# Nonlinear stability characterization of thin viscoelastic liquid films flowing down a plate moving in a vertical direction

Hung Ming Sung, Chih-Hsin Tsai, Chung-Ting Hsu

## Abstract

This project presents a nonlinear stability analysis of thin viscoelastic liquid films flowing down a plate moving in a vertical direction. The long-wave perturbation method is employed to derive the generalized kinematic equations for a free film interface. The elaborated nonlinear film flow model is solved by the method of multiple scales. The modeling results clearly indicate that both subcritical instability and supercritical stability conditions are possibly to occur in the film flow system. The effect of the down-moving motion of the vertical plate tends to enhance the stability of the film flow.

**Keywords:** viscoelastic liquid, thin film flow, long-wave perturbation, method of multiple scales.

Hung Ming Sung, Associate Professor, Department of Mechanical Engineering, HIT.
Chih-Hsin Tsai, Associate Professor, Department of Mechanical Engineering, HIT.
Chung-Ting Hsu, Lecturer, Department of Mechanical Engineering, HIT.

# 沿垂直方向移動之直立平板表面流下的黏彈性流體薄膜流的非線性穩定性分析

宋鴻明、蔡志信、徐仲亭


## 摘 要

本文針對黏彈性流體薄膜流，探討沿垂直方向移動的直立平板表面流下的薄膜流之非線性液動穩定性問題。首先使用長波微擾法推導薄膜的自由面方程式，採用多重尺度法分析薄膜流場的非線性穩定性問題。結果發現流場存在亞臨界不穩定及超臨界穩定現象，平板往下移動的效應，使系統趨於穩定。


**關鍵詞**：黏彈性流體，薄膜流，長波微擾法，多重尺度法。

宋鴻明：修平技術學院機械系副教授
蔡志信：修平技術學院機械系副教授
徐仲亭：修平技術學院機械系講師
投稿日期：970603 接受刊登日期：970704

# 1. Introduction

The stability characterization of film flows traveling down along a vertical or an inclined plate is of great importance to the quality control of many industrial products. Thus, the research effort made toward improvement on this matter has been emerged as a subject of great interest to numerous worldwide researchers in past decades. Typical application examples can be found across different industrial sectors including mechanical, chemical and nuclear engineering. It is well known that the stability controls are generally required in precision finishing processes of coating, laser cutting, and casting. Since macroscopic instability can cause disastrous conditions to film flows and thus very detrimental to the needed quality of final products, it is highly desirable to develop suitable working conditions for homogeneous film growth to adapt to various flow configurations and associated time-dependent properties.

Benney [1] investigated the nonlinear evolution equation for free surfaces of the film flows by using the method of small parameters. The solutions thus obtained were used to predict nonlinear instability conditions. However, the solutions cannot be used to predict supercritical stability since the influence of surface tension is neglected in the small-parameter modeling method. The effect of surface tension on flow stability was considered significant by Lin [2], Nakaya [3], and Krishna et al. [4]. Pumir et al. [5] further included the effect of surface tension in the film flow model and solved for the solitary wave solutions. Hwang et al. [6] showed that both the conditions of supercritical stability and sub-critical instability for a film flow system are possible to occur. Renardy et al. [7] and Tsai et al. [8] presented the work of both linear and nonlinear stability analyses for a film flow traveling down along an inclined or a vertical plate. Detailed flow analysis was found of great importance in the development of stability theory for characterizing various behaviors of film flows.

Andersson et al. [9] studied the gravity-driven flow of a viscoelastic film flow traveling down along a vertical wall. The derived analytical expression of film thickness reveals that the film thickness of a viscoelastic film can develop more rapidly than that of the Newtonian film in downstream asymptotic states. Walters [10] analyzed the motion behavior of a viscoelastic film flow that is confined in between two coaxial cylinders. Cheng et al.

[11] studied the stability of thin viscoelastic film flow traveling down along a vertical wall. The results of their studies indicate that the viscoelastic parameter indeed plays a significant role in destabilizing the film flow.

After careful literature review on the papers of thin viscoelastic film flows raveling down along a vertical plate, it was found that the stability of thin viscoelastic film flows moving along vertical plates appeared to be very important in various coating, painting, surface drawing and lubrication processes. This type of stability problems has not yet been fully explored so far in the literature. The types of stability problems are indeed of great importance for many industrial applications. In this paper, the finite-amplitude stability of a thin viscoelastic film flow traveling down along a vertical quiescent, up-moving, and down-moving plate is thoroughly investigated. The influence of the plate moving styles on the equilibrium finite amplitude is studied and characterized. Several numerical examples are presented to verify the computational results and also to illustrate the effectiveness of the proposed modeling approach.

## 2. Generalized Kinematic Equation

Fig.1 shows the configuration of a thin viscoelastic film flow traveling down along a vertically moving plate. The fluid used for study is an incompressible viscoelastic prototype that is designated as liquid $B''$ by Beard and Walters[12]. The Walters' liquid $B''$ represents an approximation to the first order in elasticity, i.e. for short or rapidly fading memory fluids. All associated physical properties and the rate of film flow are assumed to be constant (i.e. time-invariant). Based on the given assumptions, the velocity fields of the film flow can be represented by

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \tag{1}$$

$$\begin{aligned}&\rho(\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*})\\&=\rho g+\frac{\partial \tau_{x^*x^*}}{\partial x^*}+\frac{\partial \tau_{y^*x^*}}{\partial y^*}\end{aligned} \tag{2}$$

$$\begin{aligned}&\rho(\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*})\\&=\frac{\partial \tau_{x^*y^*}}{\partial x^*}+\frac{\partial \tau_{y^*y^*}}{\partial y^*}\end{aligned} \tag{3}$$

where $\rho$ is the density of the film flow. Individual stress components can be expressed in terms of velocity gradient and flow pressure as

$$\tau_{x^*x^*} = -p^* + 2\mu\frac{\partial u^*}{\partial x^*} - 2k_0[\frac{\partial^2 u^*}{\partial t^*\partial x^*} + u^*\frac{\partial^2 u^*}{\partial x^{*2}} + v^*\frac{\partial^2 u^*}{\partial x^*\partial y^*} - 2(\frac{\partial u^*}{\partial x^*})^2 - \frac{\partial u^*}{\partial y^*}(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*})] \quad (4)$$

$$\tau_{y^*y^*} = -p^* + 2\mu\frac{\partial v^*}{\partial y^*} - 2k_0[\frac{\partial^2 v^*}{\partial t^*\partial y^*} + v^*\frac{\partial^2 v^*}{\partial y^{*2}} + u^*\frac{\partial^2 v^*}{\partial x^*\partial y^*} - 2(\frac{\partial v^*}{\partial y^*})^2 - \frac{\partial v^*}{\partial x^*}(\frac{\partial v^*}{\partial x^*} + \frac{\partial u^*}{\partial y^*})] \quad (5)$$

$$\tau_{x^*y^*} = \tau_{y^*x^*} = \mu(\frac{\partial u^*}{\partial y^*} + \frac{\partial v^*}{\partial x^*}) - k_0[\frac{\partial^2 u^*}{\partial t^*\partial y^*} + \frac{\partial^2 v^*}{\partial t^*\partial x^*} + u^*(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial x^*\partial y^*}) + v^*(\frac{\partial^2 v^*}{\partial x^*\partial y^*} + \frac{\partial^2 u^*}{\partial y^{*2}}) - 2\frac{\partial u^*}{\partial y^*}\frac{\partial v^*}{\partial y^*} - 2\frac{\partial v^*}{\partial x^*}\frac{\partial u^*}{\partial x^*}] \quad (6)$$

where $u^*$ and $v^*$ are velocity components in $x^*$ and $y^*$ directions, respectively. p is the flow pressure, $\rho$ is the film density, and $\mu$ is the dynamic viscosity. The boundary conditions for the film flow system at the plate surface of $y^* = 0$ can be expressed as

$$u^* = U^* \quad (7)$$

$$v^* = 0 \quad (8)$$

where $U^*$ is the moving velocity of the vertical plate. The boundary conditions for the film flow at free surface of $y^* = h^*$ are derived based on the results given by Edwards et al. [13]. The shear stress for film flow at free surface is given as

$$\frac{\partial h^*}{\partial x^*}[1 + (\frac{\partial h^*}{\partial x^*})^2]^{-1}(\tau_{y^*y^*} - \tau_{x^*x^*}) + [1 - (\frac{\partial h^*}{\partial x^*})^2][1 + (\frac{\partial h^*}{\partial x^*})^2]^{-1}\tau_{x^*y^*} = 0 \quad (9)$$

The normal stress for film flow at free surface is given as

$$[1 + (\frac{\partial h^*}{\partial x^*})^2]^{-1}[2\tau_{x^*y^*}\frac{\partial h^*}{\partial x^*} - \tau_{y^*y^*} - \tau_{x^*x^*}(\frac{\partial h^*}{\partial x^*})^2] + S^*\{\frac{\partial^2 h^*}{\partial x^{*2}}[1 + (\frac{\partial h^*}{\partial x^*})^2]^{-3/2}\} = Pa^* \quad (10)$$

The kinematic condition that the flow velocity normal to a free surface is naught can be given as

$$\frac{\partial h^*}{\partial t^*} + \frac{\partial h^*}{\partial x^*}u^* - v^* = 0 \quad (11)$$

where $p_a^*$ is the ambient pressure, $S^*$ is the surface tension, $h^*$ is the local film thickness. The variable associated with a superscript " * " stands for a dimensional quantity. By introducing the stream function $\varphi^*$, the dimensional velocity components can now be expressed as

$$u^* = \frac{\partial \varphi^*}{\partial y^*}, \quad v^* = -\frac{\partial \varphi^*}{\partial x^*} \quad (12)$$

In order to minimize the flow variables and to simplify the analysis procedure, it is customary to define dimensionless variables as

$$x=\frac{\alpha x^*}{h_0^*},\quad y=\frac{y^*}{h_0^*},\quad t=\frac{\alpha u_0^* t^*}{h_0^*},$$
$$h=\frac{h^*}{h_0^*},\varphi=\frac{\varphi^*}{u_0^* h_0^*},p=\frac{p^*-p_a^*}{\rho u_0^{*2}}$$
$$\mathrm{Re}=\frac{u_0^* h_0^*}{\nu},S=\left(\frac{S^{*3}}{2^2\rho^3\nu^4 g}\right)^{1/3}$$
$$\alpha=\frac{2\pi h_0^*}{\lambda},u_0^*=\frac{g h_0^{*2}}{2\nu},\quad k=\frac{k_0}{\rho h_0^{*2}}$$
(13)

The moving velocity of the vertical plate can then be expressed as

$$U^*=Z u_0^* \tag{14}$$

where Z is a specific constant ratio of the plate velocity to the free stream velocity.

Since the modes of long-wavelength that gives the smallest wave number are most likely to induce flow instability for the film flow [4,5], the dimensionless wave number of the long-wavelength mode, $\alpha$, is then chosen as the perturbation parameter for variable expansion. By so doing the stream function and flow pressure can be perturbed and represented as

$$\varphi=\varphi_0+\alpha\varphi_1+O(\alpha^2) \tag{15}$$
$$p=p_0+\alpha p_1+O(\alpha^2) \tag{16}$$

In practice, the non-dimensional surface tension S is a large value. The term $\alpha^2 S$ can be treated as a quantity of zero-th order [8]. The generalized nonlinear kinematic equation can be obtained as

$$h_t+A(h)h_x+B(h)h_{xx}+C(h)h_{xxxx}+D(h)h_x^2+E(h)h_x h_{xxx}=0 \tag{17}$$

where

$$A(h)=\frac{2}{1+Z}h^2+\frac{Z}{1+Z} \tag{18}$$

$$B(h)=\alpha\,\mathrm{Re}\left[\frac{8}{15}\frac{1}{(1+Z)^2}h^6+\frac{4}{3}k\frac{1}{(1+Z)^2}h^4\right] \tag{19}$$

$$C(h)=\frac{2}{3}\mathrm{Re}^{-\frac{2}{3}}S\alpha^3(1+Z)^{-1/3}h^3 \tag{20}$$

$$D(h)=\alpha\,\mathrm{Re}\left[\frac{16}{5}\frac{1}{(1+Z)^2}h^5+\frac{16}{3}k\frac{1}{(1+Z)^2}h^3\right] \tag{21}$$

$$E(h)=2\,\mathrm{Re}^{-\frac{2}{3}}S\alpha^3(1+Z)^{-1/3}h^2 \tag{22}$$

In order to characterize more precisely the effect of vertical plate motion on the stability behaviors of a down-traveling thin film flow, a detailed numerical investigation on flow stability is carried out. Three different kinds of plate-moving styles, i.e. stationary, up-moving, and down-moving movements, for various speeds are used to characterize the behaviors of stable thin film flows traveling down along the moving plate. The flow rate of the film flow is assumed to be constant. The variations of local film thickness and the flow velocity at free surface in equilibrium are defined as

$$u_0^* = (1+Z)\left(\frac{\frac{5}{24}}{\frac{5}{24}+\frac{Z}{4}}\right)^{1/2}\overline{u}_0^* \quad (23)$$

$$h_0^* = \left(\frac{\frac{5}{24}}{\frac{5}{24}+\frac{Z}{4}}\right)^{1/4}\overline{h}_0^* \quad (24)$$

where $\overline{u}_0^*$ is the velocity at free surface for a static plate in equilibrium state, and $\overline{h}_0^*$ is the film thickness in equilibrium state when the plate is static.

## 3. Stability Analysis

The dimensionless film thickness when expressed in perturbed state can be given as

$$h(x,t) = 1+\eta(x,t) \quad (25)$$

where $\eta$ is a perturbed quantity of stationary film thickness. By inserting equation (25) into equation (17) and collecting all terms up to the order of $\eta^3$, the evolution equation of $\eta$ becomes

$$\begin{aligned}&\eta_t + A\eta_x + B\eta_{xx} + C\eta_{xxxx} + D\eta_x^2 + E\eta_x\eta_{xxx}\\ &= -[(A'\eta + \frac{A''}{2}\eta^2)\eta_x + (B'\eta + \frac{B''}{2}\eta^2)\eta_{xx} \quad (26)\\ &+ (C'\eta + \frac{C''}{2}\eta^2)\eta_{xxxx} + (D + D'\eta)\eta_x^2\\ &+ (E + E'\eta)\eta_x\eta_{xxx}] + O(\eta^4)\end{aligned}$$

where all the values of A, B, C, D, E and their derivatives are evaluated at the dimensionless film height of the film h=1.

### *3.1.Linear stability analysis*

To characterize the linear behaviors of the film flow, the nonlinear terms in equation (26) are assumed insignificant and can be neglected to obtain the linearized equation

$$\eta_t + A\eta_x + B\eta_{xx} + C\eta_{xxxx} = 0 \quad (27)$$

The normal mode analysis [16] can be performed by assuming that

$$\eta = a\exp[i(x-dt)] + c.c. \quad (28)$$

where $a$ is the perturbed wave amplitude, and c.c. is the associated complex conjugate counterpart. The complex wave celerity, d, can be expressed as

$$d = d_r + id_i = A + i(B-C) \quad (29)$$

where $d_r$ is the linear wave speed, and $d_i$ is the linear growth rate of the wave amplitudes. The flow is linearly unstable supercritical if $d_i > 0$, and is linearly stable sub-critical if $d_i < 0$.

### *3.2. Nonlinear stability analysis*

In order to characterize the nonlinear behaviors of thin film flows, the method of multiple scales is employed here and the resulting Ginzburg-Landau equation [14] can be derived as

$$\frac{\partial a}{\partial t_2} + D_1\frac{\partial^2 a}{\partial x_1^2} - \varepsilon^{-2}d_i a + (E_1 + iF_1)a^2\overline{a} = 0 \quad (30)$$

where

$$e = e_r + ie_i = \frac{B' - C' + D - E}{16C - 4B} + i\frac{-A'}{16C - 4B} \quad (31)$$

$$D_1 = B - 6C \quad (32)$$

$$E_1 = (-5B' + 17C' + 4D - 10E)e_r - A'e_i - \frac{3}{2}B'' + \frac{3}{2}C'' + D' - E' \tag{33}$$

$$F_1 = (-5B' + 17C' + 4D - 10E)e_i + A'e_r + \frac{1}{2}A'' \tag{34}$$

In the above expressions, the overhead bar denotes the complex conjugate counterpart of the underlying variables. Eq. (30) can be used to characterize the weak nonlinear behaviors of the traveling film flow. The solution of the exponential form is assumed and given as

$$a = a_0 \exp[-ib(t_2)t_2] \tag{35}$$

By substituting the above expression into Eq. (30), one can obtain

$$\frac{\partial a_0}{\partial t_2} = (\varepsilon^{-2} d_i - E_1 a_0^2) a_0 \tag{36}$$

$$\frac{\partial [b(t_2)t_2]}{\partial t_2} = F_1 a_0^2 \tag{37}$$

The condition for a supercritical stable region to exist in the linearly unstable region $(d_i > 0)$ is $E_1 > 0$. Thus, the associated wave amplitude $\varepsilon a_0$ becomes

$$\varepsilon a_0 = \sqrt{\frac{d_i}{E_1}} \tag{38}$$

The nonlinear wave speed is now derived and given as

$$Nc_r = \varepsilon^2 b = d_r + d_i \left(\frac{F_1}{E_1}\right) \tag{39}$$

On the other hand, the condition for the flow behavior of sub-critical instability in the linearly stable region $(d_i < 0)$ is $E_1 < 0$. The threshold amplitude of the wave is denoted as $\varepsilon a_0$. The sub-critical stable region can only be found for the condition of $E_1 > 0$. The neutral stability curve can be derived and plotted for the condition of $E_1 = 0$. Based on the above discussion, it is obvious that the Ginzburg-Landau equation can be used to characterize various flow states.

## 4. Numerical Illustrations and Discussions

A numerical example is presented here to illustrate the effectiveness of the proposed modeling approach for characterizing the thin viscoelastic film flow traveling down along a vertically moving plate. In order to reliably verify the results of theoretic derivation, a finite amplitude perturbation apparatus is used to numerically generate the needed perturbation parameters for linear stability analyses. It is obvious from the nonlinear kinematic equation that the stability of a thin-film flow is closely related and can be characterized by several flow variables including Reynolds number, Re, velocity ratio of the plate to free stream, Z, viscoelastic parameter, k, and dimensionless perturbation wave number, $\alpha$ . Some important features appeared in modeling results are carefully extracted and used to compare with some conclusive results given in the literature.

Fig. 1 shows the schematic diagram of a thin viscoelastic film flow traveling down along a vertically down-moving upright plate. Physical parameters that are selected for study include (1) Reynolds numbers ranging from 0 to 15, (2) the dimensionless perturbation wave numbers ranging from 0 to 0.12, (3) the value of viscoelastic parameter is given as 0.125[11], and (4) the velocity ratios Z for use in this study include –0.42, -0.32, -0.18, 0, 0.23, 0.51, 0.85. A constant dimensionless surface tension value is given for computation to enable the study of film flow stability behaviors for different plate-moving conditions of moving-up (Z = -0.42, -0.32, -0.18), stationary (Z = 0), and moving-down (Z = 0.23, 0.51, 0.85). In other words, S is selected as 6173.5 [11]. This value is selected here for study mainly for comparing the final result with data given in the literature. It is found that the results obtained by using the proposed method for the thin viscoelastic film flow traveling down along a stationary vertical plate (i.e. Z = 0) agree well with those data given by Cheng et al. [11].

As the perturbed wave grows to finite amplitude, the linear stability theory is no longer valid for accurate prediction of flow behaviors. The nonlinear stability analysis will have to be used to study the effect of finite amplitude disturbances on the stability behaviors of the flow in the linearly stable region. In other words, the nonlinear, instead of linear, flow stability theory will have to be used to characterize the behavior of sub-critical instability in the linearly stable region. By using the nonlinear flow stability theory, one can characterize two different possible flow behaviors including (1) subsequent nonlinear evolution of disturbances in the linearly unstable region may be redeveloped to become a new equilibrium state of finite amplitudes (i.e. supercritical stability), or (2) the flow may become unstable eventually. The flow instability in the linearly stable region, as named sub-critical instability, can be easily realized by setting the variable $E_1$ in Eq. (36) to a negative value. In other words, if $E_1$ in Eq. (36) is a negative value, the amplitude of disturbed waves in the linearly stable region is possibly to develop into a unstable state. This is completely different from that of the prediction obtained by the linear stability analysis that gives the result of strict stability. The flow stability in the linearly unstable region, as named supercritical stability, can be easily realized by setting the variable $E_1$ in Eq. (36) to a positive value. In other words, if $E_1$ in Eq. (36) is a positive value, the amplitude of disturbed wave in the linearly unstable region may be redeveloped to a new equilibrium state of finite amplitudes. The nonlinear neutral stability curves can be obtained by simultaneously setting $d_i = 0$ in Eq. (29) and $E_1 = 0$ in Eq. (33). The areas near the neutral stability curves in Figs. 2(a)-2(e) reveal that several different flow conditions including sub-critical instability $(d_i < 0,\ E_1 < 0)$, sub-critical stability $(d_i < 0,\ E_1 > 0)$, supercritical stability $(d_i > 0, E_1 > 0)$, and explosive supercritical instability $(d_i > 0,\ E_1 < 0)$, are possibly to occur at different velocity ratios, Z.

Fig. 2(a) shows the nonlinear neutral stability curves of the film flow traveling down along a stationary (i.e. Z=0) vertical plate. The nonlinear neutral stability curves of the film flow traveling down along a vertical-moving plate for velocity ratios of Z = 0.23, 0.51, -0.18, -0.32 are computed and presented in Figs. 2(b)-2(e). The results indicate that when the down-moving plate velocity increases, the areas for both regions of sub-critical instability and sub-critical stability increase gradually. It also deserves noting that when the down-moving plate velocity increases, the area for the region of explosive supercritical instability decreases gradually, and the region of supercritical stability presents no obvious change. On the other hand, when up-moving plate velocity increases, the areas for both regions of sub-critical instability and sub-critical stability decrease gradually, the area for the region of explosive supercritical instability increases gradually, and the region of supercritical stability presents no obvious change. It is clear that in Figs. 2(a)-2(e) the areas of shaded regions (including sub-critical instability and explosive supercritical instability) decrease as the down-moving plate velocity increases. Also, the shaded areas increase as the up-moving plate velocity increases.

Fig. 3 shows the nonlinear threshold amplitude curves of the perturbed wave in the sub-critical unstable region for various perturbed wave numbers at various velocity ratios and $\overline{\mathrm{Re}} = 5$. It is found that the threshold values of the wave amplitude curves in the sub-critical unstable region increase as the down-moving plate velocity increases. In such a situation, the film flow will become stable. That is to say, if the initial finite amplitude disturbance is less than the threshold amplitude, the system will become conditionally stable. On the other hand, if the initial finite amplitude disturbance is greater than the threshold amplitude, the system will become explosively unstable. It is also found that the threshold values of the wave amplitude curves in the sub-critical unstable region decrease as the up-moving plate velocity increases. In such a situation, the film flow will become unstable.

Fig.4 shows the nonlinear threshold amplitude of the perturbed wave in the supercritical stable region for various perturbed wave numbers at different velocity ratios and $\overline{\mathrm{Re}} = 10$. It is found that the values of the threshold wave amplitude in the supercritical stable region decrease as the down-moving plate velocity increases. Also, the values of the threshold wave amplitude in the supercritical stable region increase as the up-moving plate velocity increases. The wave speed in Eq. (29) that is predicted by using the linear model is a constant value for all wave number. However, the wave speed in Eq. (39) that is predicted by using the nonlinear model is a function of wave number, Reynolds number, and velocity ratio Z. Fig. 5 shows the nonlinear wave speeds in the supercritical stable region under various perturbed wave numbers at different velocity ratios and $\overline{\mathrm{Re}} = 10$. It is found that the nonlinear wave speed in the supercritical stability region decreases as the down-moving plate velocity increases. Also, the nonlinear wave speed in the supercritical stability region increases as the up-moving plate velocity increases. It becomes quite obvious from Figs. 4-5 that the film flow system becomes more stable as the down-moving plate velocity increases. Also, the film flow system becomes more unstable as the up-moving plate velocity increases.

## 5. Conclusion

The nonlinear stability of a thin viscoelastic film flow traveling down along a vertical plate under three different plate moving conditions is investigated by using the method of long-wave perturbation. The generalized nonlinear kinematic equations of the film flow at the interface of free surface is derived and numerically estimated to characterize the behaviors of flow stability. Based on the results of numerical modeling, several conclusions can be drawn as follows:

1.The results of nonlinear modeling

analyses indicate that when the down-moving plate velocity increases, the areas for both regions of sub-critical instability and sub-critical stability increase gradually, the area for the region of explosive supercritical instability decreases gradually, and the region of supercritical stability presents no obvious change. It is found that the threshold amplitude of the perturbed wave in the sub-critical unstable region increases as the down-moving plate velocity increases. The values of the threshold wave amplitude as well as nonlinear wave speed of the flow in the supercritical stable region decrease when the down-moving plate velocity increases. On the other hand, when the up-moving plate velocity increases, the areas for both regions of sub-critical instability and sub-critical stability decrease gradually, the area for the region of explosive supercritical instability increases gradually, and the region of supercritical stability presents no obvious change. It is found that the threshold amplitude of the perturbed wave in the sub-critical unstable region decreases and both of the values of the threshold wave amplitude and nonlinear wave speed of the flow in the supercritical stable region increase as the up-moving plate velocity increases.

2.The stability behaviors of a thin viscoelastic film flow are significantly affected by the moving style. It is conclusive that the down-moving motion of the vertical plate tends to enhance the stability of the down-traveling film flow on the plate.

## References

[1]D.J. Benney, “Long waves on liquid film”, J. Math. Phys. 45 (1966) pp.150-155.

[2]Lin S. P. “Finite amplitude side-band stability of a viscous film,” J. Fluid Mech. 63 (1974), pp.417-429.

[3]Nakaya C. “Equilibrium state of periodic waves on the fluid film down a vertical wall,” J. Phys. Soc. Japan 36 (1974), pp.921-926.

[4]Krishna M. V. G. and Lin S. P. “Nonlinear stability of a viscous film with respect to three-dimensional side-band disturbance,” Physics of Fluids 20 (1977), pp.1039-1044.

[5]A. Pumir, P. Manneville and Y. Pomeau, “On solitary waves running down on inclined plane”, J. Fluid Mech. 135 (1983), pp.27-50.

[6]C.C. Hwang and C.I. Weng, “Finite-amplitude stability analysis of liquid films down a vertical wall with and without interfacial phase change”,

International J. of Engineering Science 13 (1987), pp.803-814.

[7]Y. Renardy and S.M. Sun, "Stability of a layer of viscous magnetic fluid flow down an inclined plane", Phys. Fluids 6 (1994), pp.3235-3246

[8]J.S. Tsai, C.I. Hung and C.K. Chen, "Nonlinear hydromagnetic stability analysis of condensation film flow down a vertical plate", Acta Mechanica 118 (1996), pp.197-212.

[9]Andersson H. I. and Dahi E. N. "Gravity-driven flow of a viscoelastic liquid film along a vertical wall," J. Phys. D: Appl. Phys. 32 (1999), pp.1557-1562.

[10]Walters K. "The motion of an elastic-viscous liquid contained between coaxial cylinders," Quart. J. Mech. Appl. Math. 13 (1960), pp.444-461.

[11]Cheng P. J., Lai H. Y., Chen C. K. "Stability analysis of thin viscoelastic liquid film flowing down on a vertical wall," J. Phys. D: Appl. Phys., Vol. 33, No. 14 (2000), pp.1674-1682.

[12]Beard D.W. and Walters K. "Elastico-viscous boundary-layer flow I. Two dimensional flow near a stagnation point," Proc. Comb. Phil. Soc. 60 (1964), pp.667-674.

[13]Edwards D. A., Brenner H. and Wasan D. T., Interfacial transport processes and rheology. Butterworth-Heinemann (Boston, 1991).

[14]V.L. Ginzburg and L.D. Landau, Theory of superconductivity, *J. Exptl. Theoret.Phys.* (USSR) 20 (1950), pp. 1064-1082.

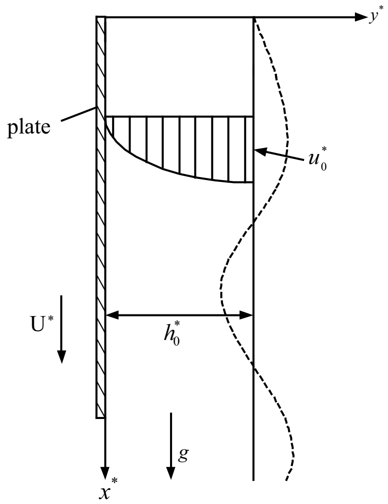


Fig. 1 Schematic diagram of a thin viscoelastic film flow traveling down along a vertically moving upright plate

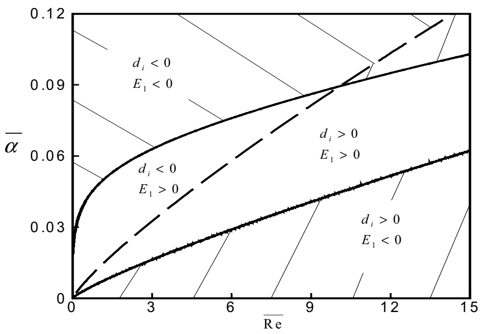


Fig. 2(a) Nonlinear neutral stability curves of the film flowfor various $\overline{\alpha}$ and $\overline{\text{Re}}$ at Z=0

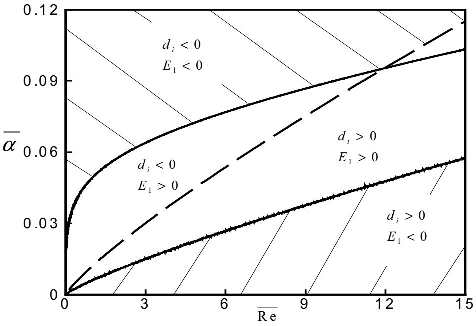


Fig. 2(b) Nonlinear neutral stability curves of the film flow for various $\overline{\alpha}$ and $\overline{\text{Re}}$ at Z=0.23

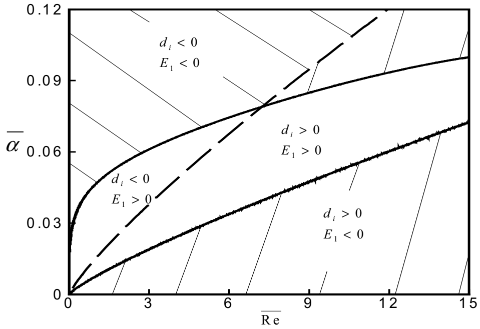


Fig. 2(e) Nonlinear neutral stability curves of the film flow for various $\overline{\alpha}$ and $\overline{\text{Re}}$ at Z=-0.32

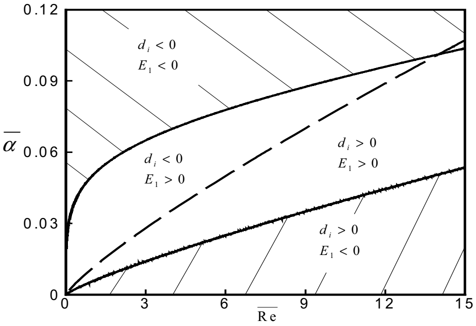


Fig. 2(c) Nonlinear neutral stability curves of the film flow for various $\overline{\alpha}$ and $\overline{\text{Re}}$ at Z=0.51

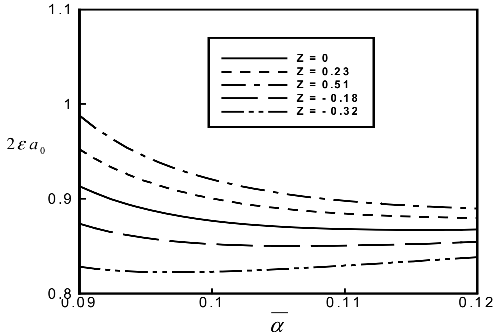


Fig. 3 Nonlinear threshold finite wave amplitude in the sub-critical unstable region for various Z and $\overline{\alpha}$ at $\overline{\text{Re}} = 5$

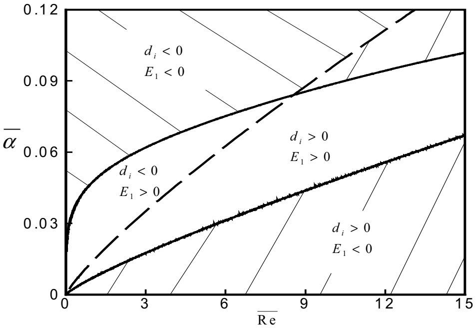


Fig. 2(d) Nonlinear neutral stability curves of the film flow for various $\overline{\alpha}$ and $\overline{\text{Re}}$ at Z=-0.18

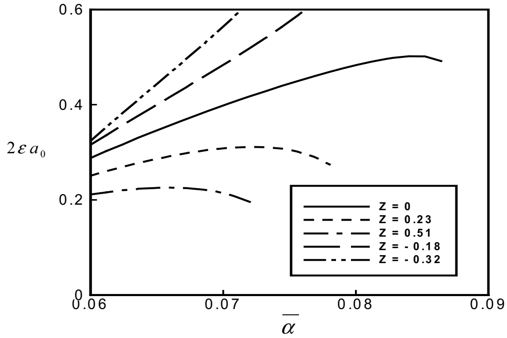


Fig. 4 Nonlinear threshold finite wave amplitude in the supercritical stable region for various Z and $\overline{\alpha}$ at $\overline{\text{Re}} = 10$

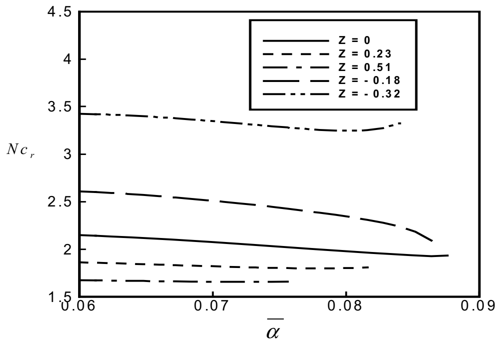


Fig. 5 Nonlinear wave speeds of the film flow in the supercritical stable region for various Z and $\bar{\alpha}$ at $\overline{\text{Re}} = 10$