行政院國家科學委員會專題研究計畫成果報告

對多模態磁浮軸承轉子系統具性能強健性 PID 控制器設計

Design of the PID Performance-Robustness Controller for the Multi-Mode Rotor System with Magnetic Bearings

計畫編號：NSC 90－2213－E－164－002

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主持人：許耿禎 修平技術學院電機工程系

E-Mail:gjsheu@mail.hit.edu.tw

計畫參與人員：呂奇璜、陳昭元 修平技術學院電機工程系

一、中文摘要

本計劃針對多目標轉子系統控制器設計，提出一具性能強健性及實現性的控制器設計法則，此法則結合了品質控制中的田口式實驗法及工業上最常應用的 PID 控制法則，透過簡單及快速的直交表實驗及驗證後，可決定感測器/致動器的最佳位置及其回饋增益值，本文最後以一具多模態的轉子系統為例，證明本控制器設計法則不但能滿足性統穩定性，亦能整合具有性能強健性及易實現性的特點。

關鍵詞：田口式實驗法、性能強健性、多模態轉子系統、PID 控制器設計

Abstract

This project presents a controller design methodology for the optimal design of sensor/actuator location and feedback gains. The methodology combines Taguchi’s method in quality control engineering and conventional PID control such that their advantages in implementation feasibility and performance-robustness can be integrated together. By using the location of the magnetic bearing and PD feedback gains as design parameters for the multi-mode rotor system with a magnetic bearing, the controller can be determined by a small number of matrix experiments to achieve the best system performance.

Keywords: Taguchi’s method, performance-robustness, multi-mode rotor system, PID controller design

二、緣由與目的

LQR algorithms were recently widely applied to rotor systems with a magnetic bearing. However, the controller order is so high that prevents any feasible implementation. Even if implementable, it is optimal or near optimal only to the rotor system model of preselected sensor/actuator location at a specific operating speed. However, structure vibration suppression depends not only on control law design but also on sensor/actuator locations. Furthermore, full-state measurement in LQR is difficult, if not impossible, and the choice of weighting matrices Q and R has often plagued many applications. Their requirement of either on-line state estimation or observer also poses an insurmountable task in the controller design of rotor systems. In contrast, traditional PD controllers have been considered preferable over LQR-based algorithms in some industrial applications.

This project will present a new methodology of controller design for the vibration control of multi-mode rotor system with a magnetic bearing. The methodology combines the experimental design method originated from Taguchi's method in quality control engineering and the conventional PID control such that their advantages in implementation feasibility and performance-robustness can be integrated together. Taguchi's method is an engineering methodology for improving productivity during research and development phases. It is to improve product quality by minimizing the effect of design and process uncertainties but without eliminating the causes, hence also called *robust design*. For the vibration suppression of rotor systems, the controller design is to search for a design parameter set, including the location of the magnetic bearing, the displacement feedback gains and the velocity feedback gains within their feasible domain and saturatuion limits, such that the closed loop system performance within a desired operation range can be achieved.

三、結果與討論

The rotor-bearing system can be modeled as a system of rigid disks, flexible shaft elements with distributed mass and stiffness, and discrete bearings as shown in Fig.1. The linearized second-order differential equations can be written as:

$$[M]\{\ddot{p}\} + ([C] - \Omega[G])\{\dot{p}\} + [K]\{p\} = [B]\{u\} + \{Q\}$$

$$\{p\} = \{V_1, W_1, B_1, \Gamma_1, \cdots, V_{(Ne+1)}, W_{(Ne+1)}, B_{(Ne+1)}, \Gamma_{(Ne+1)}\}$$

Based on Maxwell's law with the small oscillation assumption, the closed loop rotor-bearing system with a magnetic bearing is written by

$$[M]\{\ddot{p}\} + ([C] - \Omega[G])\{\dot{p}\} + ([K] + [BK])\{p\} = -K_i[B]\{i\} + \{Q\}$$

where

$$[B] = \begin{bmatrix} 0 & \cdots 1 & 0 \cdots & 0 \\ 0 & \cdots 0 & 1 \cdots & 0 \end{bmatrix}^T_{4(Ne+1)x2}$$

$$[BK] = diag(0, \cdots, K_h, K_h, \cdots, 0)_{4(Ne+1)x4(Ne+1)}$$

$$\{i\} = \{i_v, i_w\}^T$$

For a magnetic bearing in collocated feedback configuration, the control currents can be set in the following form,

$$\begin{Bmatrix} i_v \\ i_w \end{Bmatrix} = -\begin{bmatrix} D_2 & D_6 \\ D_6 & D_4 \end{bmatrix} \begin{Bmatrix} \dot{V}_l \\ \dot{W}_l \end{Bmatrix} - \begin{bmatrix} D_1 & D_5 \\ D_5 & D_3 \end{bmatrix} \begin{Bmatrix} V_l \\ W_l \end{Bmatrix}$$

For the rotor system as shown in Fig. 1, there are three critical speeds at 644, 872, and 1320 (*rad/sec*) the desired operation range up to 2000 (*rad/sec*). Figure 2 illustrates the first three forward modes in circular motion. The objective is to place a magnetic bearing with suitable PD feedback gains to minimize the structural vibration within the desired operation range. The possible locations for magnetic bearing are at node 2, 7, 8, 9, 10 and 11, thus the design parameter of location has six levels. A standard L_{18} (6^1x3^6) array is preferred for studying the controller design.

Consider the quality loss Q_L as the summation of square of infinity norm of the unbalanced response from the first three modes. Figure 2 indicates that the anti-node amplitudes of the open loop system at node 1, 8 and 10 can be employed as the proper index to measure system performance. The quality loss Q_L is then defined by

$$Q_L = \frac{1}{3}\left[r_1^2(\bmod e2) + r_8^2(\bmod e3) + r_{10}^2(\bmod e1)\right]$$

And it is expressed in decibels by

$$\eta = 10\log_{10}\left(\frac{1}{Q_L}\right)$$

where η is called the signal-to-noise ratio, *SNR*. Since *log* is a momotonically increasing function, minimizing Q_L is equivalent to maximizing η.

The quality loss factor and the signal noise ratio of each experiment can be calculated and shown in Table 1. Table 2 lists the mean effects of each design parameter in which the optimal level with the highest η is marked. The matrix experiments predict that the optimal design is by placing the magnetic bearing at level 3 with feedback gains D_i at level 2, 3, 3, 3, 2, and 1, respectively; i.e., by placing the magnetic bearing at node 8 with the feedback gains of D_1=4, D_2=10, D_3 =7, D_4 =10, D_5 =1, and D_6 =0. Note that the optimal set does not necessarily correspond to any row in the matrix experiment. The design parameter of magnetic bearing location is shown to be most sensitive (93.46%) to system performance, indicating that the magnetic bearing location influences system performance most. Followed by the velocity feedback gains D_2 and D_4 of 2.43% and 1.93%, respectively.

The unbalanced response of the open loop system (Q_L =6.961x10^{-10}) is shown in Fig. 3. Figure 4 shows the unbalanced response of experiment 2 (Q_L =5.493x 10^{-10}). There is virtually no vibration suppression in mode 1 and 3 because the magnetic bearing at node 2 is close to the anti-node of mode 2. Compared with open loop system, experiment 2 can improve the system performance by 21.2%. The unbalanced response of experiment 16 (Q_L =6.628x10^{-10}) is also plotted in Fig. 5, in which the magnetic bearing control at node 11 has little influence on mode 2 and 3 for the same reason. The unbalanced response of the optimal design (Q_L =2.931x10^{-10}) is plotted in Fig. 6. Compared with that of the open loop system, experiment 2 and 16, the controller can improve the performance by as much as 57.9%, and it is shown effective for vibration suppression of all modes within the 2000 *rad/sec* operation range.

四、計畫成果自評

A new controller design methodology by Taguchi's method is developed for the vibration suppression of rotor systems with a magnetic bearing. The methodology integrates the Taguchi's method with conventional PD control technique such that their advantages in implementation feasibility and performance-robustness can be integrated.

Compared with other LQR-based controller designs, the controller is shown to be: (1) effective in meeting the performance requirement, (2) robust and adaptable to system with operating speed variations, (3) the controller order is smaller and feasible in implementation for no full state measurement nor state estimation is required, and (4) one can achieve the desired performance of vibration suppression by tailoring Q_L for different condition and emphasis.

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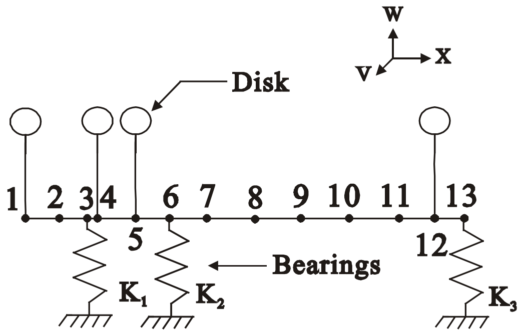

Fig. 1 Schematic diagram of the rotor system

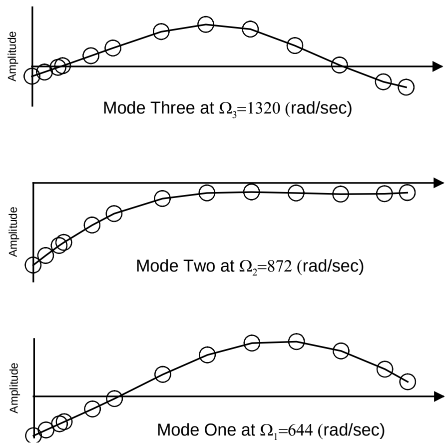

Fig. 2 The first three forward modes.

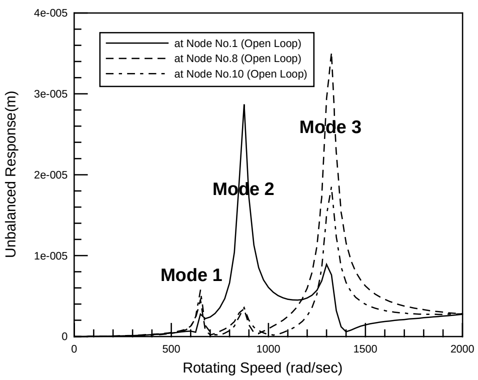

Fig. 3 Unbalanced response of the open loop system.

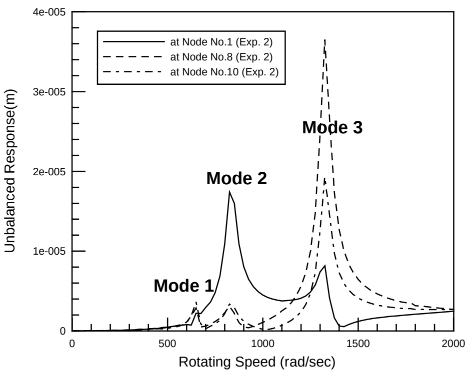

Fig. 4 Unbalanced response of experiment 2.

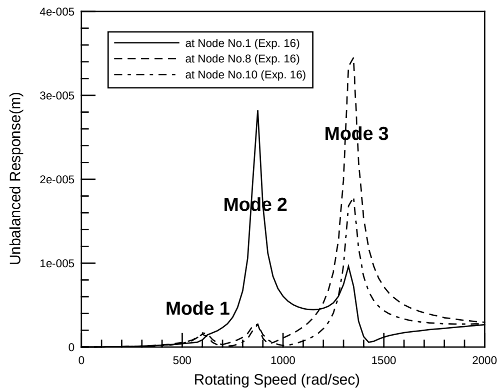

Fig. 5 Unbalanced response of experiment 16.

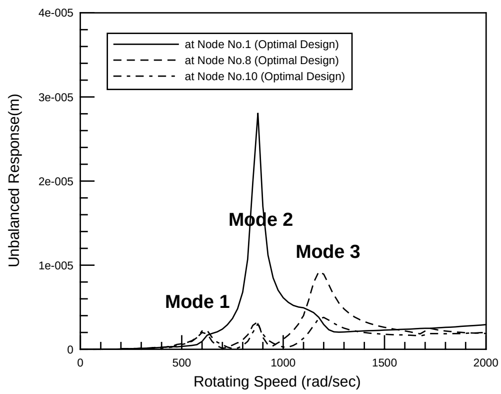

Fig. 6 Unbalanced response of the optimal design.

Table 2. *SNR*(*dB*) of each design parameter and level

Level	S/A	D_1	D_2	D_3	D_4	D_5	D_6
1	92.552	93.529	93.426	93.682	93.498	93.570	$\boxed{93.714}$
2	94.583	$\boxed{93.808}$	93.691	93.571	93.608	$\boxed{93.783}$	93.589
3	$\boxed{94.804}$	93.664	$\boxed{93.884}$	$\boxed{93.746}$	$\boxed{93.895}$	93.647	93.697
4	94.713						
5	93.600						
6	91.750						
Max-Min	3.054	0.279	0.458	0.175	0.397	0.213	0.125

($\boxed{}$: maximum *SNR* in each column)

Table 1. Test Results of the L_{18} orthogonal array (unit: x10^{-5}m)

Exp. No.	Stability	r_{10}(mode 1)	r_1 (mode 2)	r_8 (mode 3)	Q_L(x10^{-10})	*SNR(dB)*
Open Loop	Stable	0.571	2.87	3.510	6.961	91.573
1	Stable	0.361	2.29	3.700	6.355	91.969
2	Stable	0.361	1.74	3.650	5.493	92.602
3	Stable	0.368	1.34	3.580	4.916	93.084
4	Stable	0.535	2.78	1.940	3.926	94.060
5	Stable	0.413	2.73	1.360	3.158	95.006
6	Stable	0.460	2.75	1.560	3.403	94.682
7	Stable	0.270	2.83	1.110	3.105	95.080
8	Stable	0.310	2.84	1.220	3.217	94.926
9	Stable	0.400	2.85	1.610	3.625	94.407
10	Stable	0.216	2.85	1.170	3.179	94.977
11	Stable	0.282	2.86	1.330	3.343	94.760
12	Stable	0.423	2.86	1.590	3.629	94.402
13	Stable	0.376	2.86	2.390	4.478	93.300
14	Stable	0.203	2.85	2.050	4.122	93.850
15	Stable	0.252	2.85	2.180	4.313	93.652
16	Stable	0.169	2.82	3.450	6.628	91.786
17	Stable	0.328	2.87	3.450	6.749	91.708
18	Stable	0.236	2.84	3.450	6.675	91.756
Optimal Design	Stable	0.221	2.82	0.889	2.931	95.331

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