行政院國家科學委員會專題研究計畫　成果報告

針對高速自旋主軸系統設計具溢出穩定性最佳控制器

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執行期間：91 年 08 月 01 日至 92 年 07 月 31 日
執行單位：修平技術學院電機工程系

計畫主持人：許耿禎

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The Optimal Controller Design of a High Spinning Shaft with Spillover Stabilization

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一、中文摘要

本計劃應用 Rayleigh 樑理論並包含轉動慣量及陀螺效應的連續分佈參數模式，來建立高速自旋主軸系統模型，並求出包括：迴轉速度、臨界轉速及不平衡響應等動態響應正確解析解，並將其結果應用於最佳振動控制器設計上。然因受限於控制器的階數不能太高，系統必須做適度的降階，不過所忽略的殘餘模態若無經過適度的處理，將會產生溢出不穩定性。本計劃可針對殘餘模態所產生之溢出不穩定問題加入穩定性探討，以確保本計劃所發展的最佳控制器設計，不僅能確保系統之絕對穩定性，並同時能達到最佳減振效果。

關鍵詞：臨界轉速、共振模態、溢出穩定性、最佳控制器設計

Abstract

It has been shown that a spinning shaft has only finite number of critical speeds and the precessional modes when the whirl ratio $\lambda > 1/2$. The system's unbalanced response can therefore be expressed by the finite precessional modes and the corresponding generalized coordinates. This project presents a spillover stabilizable controller design for optimal sensor/actuator location and feedback gain such that the steady state unbalanced response can be minimized. Under controller order constraint when only part of the precessional modes are included in the controller design, the spillover from the remaining residual modes can be evaluated for system stability.

Keywords: critical speeds, precessional modes, spillover stabilization, optimal controller design

二、緣由與目的

One of the major challenges in structural control is to apply successfully and confidently the control law derived from a reduced-order model to the engineering system of much higher order. Structural control systems usually require a large number of vibration modes to describe its dynamics, but the controller is implemented only for a few vibration modes, often termed *primary modes*. It is known that spillover—the observation spillover that entails the contamination of sensor output through the presence of residual mode dynamics and the control spillover in which the residual modes are excited by feedback control—are inherent in controller design of reduced-order model.

A recent analytical study (Yang and Sheu, 1999) showed that the unbalanced response of a spinning shaft can be written analytically by a finite number of precessional modes and the corresponding generalized coordinates when $\lambda > 1/2$. Because the number of critical speeds and the precessional modes are finite, all can be included in the controller design. Spillover instability due to residual modes often seen in vibration control of flexible systems can then be prevented. However, the controller based on a full order model may often be difficult, if not impossible, to realize in practice. This project aims at evaluating the spillover, if any, of a system under controller order constraint where only part of the precessional modes within a desired operation range can be considered. The design of a spillover stabilizable controller is necessary such that the steady state unbalanced response within the operation range can be minimized.

三、結果與討論

Consider of a spinning circular shaft modeled by a Rayleigh beam with rotary inertia and gyroscopic effects as shown in Fig. 1.The EOM of a spinning shaft under control input can be rewritten analytically in a matrix form by

$$[\mathbf{M}]\{\ddot{\mathbf{q}}(\tau)\} + ([\mathbf{D}] + i\Omega[\mathbf{G}])\{\dot{\mathbf{q}}(\tau)\} + [\mathbf{K}]\{\mathbf{q}(\tau)\} = [\mathbf{B}(\zeta_{\mathbf{a}})]\{\mathbf{f}(\tau)\} + \{\mathbf{N}(\tau)\}, \quad (1)$$

where the generalized matrices [**M**], [**D**], and [**K**] are the diagonal, symmetric, positive definite mass, damping, and stiffness matrices with dimension $n_{cr} \times n_{cr}$, respectively. [**G**] is the gyroscopic matrix of same dimension. $[\mathbf{B}(\zeta_{\mathbf{a}})]$ is the control influence matrix, $\{\mathbf{f}(\tau)\}$ is the control input vector, and $\{\mathbf{N}(\tau)\}$ is the unbalanced force vector.

For a spinning shaft under velocity feedback as the control force to suppress the unbalanced vibration with r-measurement at ζ_s, the control input becomes

$$\{\mathbf{f}(\tau)\} = -[\mathbf{g}][\mathbf{C}(\zeta_{\mathbf{s}})]\{\dot{\mathbf{q}}(\tau)\} \quad (2)$$

where $[\mathbf{C}(\zeta_{\mathbf{s}})]$ is the sensor distribution matrix,

$$[\mathbf{C}(\zeta_{\mathbf{s}})] = [\phi_j(\zeta_{sm})],\ j = 1, \ldots, n_{cr} \text{ and } m = 1, \ldots, r \quad (3)$$

and [**g**] is an $n \times r$ constant gain matrix. Therefore, the closed loop system becomes

$$[\mathbf{M}]\{\ddot{\mathbf{q}}(\tau)\} + ([\mathbf{D}] + [\mathbf{B}(\zeta_{\mathbf{a}})][\mathbf{g}][\mathbf{C}(\zeta_{\mathbf{s}})] + i\Omega[\mathbf{G}])\{\dot{\mathbf{q}}(\tau)\} + [\mathbf{K}]\{\mathbf{q}(\tau)\} = \{\mathbf{N}(\tau)\}. \quad (4)$$

In the case of noncollocated sensor/actuator, $[\mathbf{B}(\zeta_{\mathbf{a}})]$ and $[\mathbf{C}(\zeta_{\mathbf{s}})]$ may not necessarily be in the same column space so that the closed loop system is no longer guaranteed stable. The control-induced damping matrix can then be written into a symmetric matrix

$$[\mathbf{D}_{\mathbf{c}}] = \frac{1}{2}\{[\mathbf{B}(\zeta_{\mathbf{a}})][\mathbf{g}][\mathbf{C}(\zeta_{\mathbf{s}})] + ([\mathbf{B}(\zeta_{\mathbf{a}})][\mathbf{g}][\mathbf{C}(\zeta_{\mathbf{s}})])^T\} \quad (5)$$

and a skew symmetric matrix

$$[\mathbf{G}_{\mathbf{c}}] = \frac{1}{2}\{[\mathbf{B}(\zeta_{\mathbf{a}})][\mathbf{g}][\mathbf{C}(\zeta_{\mathbf{s}})] - ([\mathbf{B}(\zeta_{\mathbf{a}})][\mathbf{g}][\mathbf{C}(\zeta_{\mathbf{s}})])^T\} \quad (6)$$

that represent the damping and the gyroscopic effect from feedback, respectively. The closed loop system becomes

$$[\mathbf{M}]\{\ddot{\mathbf{q}}(\tau)\} + ([\mathbf{D}] + [\mathbf{D}_{\mathbf{c}}] + [\mathbf{G}_{\mathbf{c}}] + i\Omega[\mathbf{G}])\{\dot{\mathbf{q}}(\tau)\} + [\mathbf{K}]\{\mathbf{q}(\tau)\} = \{\mathbf{N}(\tau)\}. \quad (7)$$

By the generalized Kelvin-Tait-Chetaev theorem (Yang, 1993), the system is asymptotically stable if $([\mathbf{D}] + [\mathbf{D}_{\mathbf{c}}])$ is positive definite, independent of [**G**] or [$\mathbf{G_c}$].

The steady state unbalanced response $U(\zeta, \tau)$ can be obtained explicitly as

$$U(\zeta, \tau) = [\Phi(\zeta)]^T\{\mathbf{q}(\zeta)\} = |U(\zeta)|\,\mathrm{e}^{i(\Omega\tau - \theta)}, \quad (8)$$

where $|U(\zeta)|$ is the vibration amplitude, and θ is the phase lag induced by feedback control. The controller design is to determine the optimal sensor location ζ_s, actuator location ζ_a, and feedback gain $[\mathbf{g}]$ such that the steady state unbalanced vibration can be minimized.

The measure of vibration suppression performance can be evaluated by the integrating the steady state vibration amplitude $|U(\zeta)|$ over the shaft length

$$W(\Omega, \zeta_s, \zeta_a, [\mathbf{g}]) = \int_0^1 |U(\zeta)|^2 d\zeta \qquad (9)$$

The performance index depends on the spinning speed (Ω), the location of sensor/actuator (ζ_s, ζ_a), and the feedback gain ($[\mathbf{g}]$). For a given spinning speed, the optimal sensor/actuator location (ζ_s^*, ζ_a^*) and feedback gain $([\mathbf{g}^*])$ can be obtained by the optimization problem

$$\min_{\zeta_s, \zeta_a, [g]} W(\Omega, \zeta_s, \zeta_a, [g]), \qquad (10)$$

subject to the constraints

(1) stability criteria $([\mathbf{D}] + [\mathbf{D_c}]) > 0,$

(2) admissible region $0 \le \zeta_s \le 1, \quad 0 \le \zeta_a \le 1,$

(3) saturation limit $0 \le g_{ij} \le 1.$

The optimal sensor/actuator location will change with the spinning speed because of the change of the anti-node(s) location. The above optimal control for a spinning shaft is developed for either a part or a full order model where the fisrt or all critical speeds and precassional modes are included in the controller design. The precessional modes employed in the in the reduced model for the controller design are termed the *primary modes* and the truncated modes are termed *residual modes*. If only part of the precessional modes within a desired operation range are included in the controller design, then the spillover effects of residual modes will have to be evaluated. Equation (7) can be decomposed into

$$[\mathbf{M}_p]\{\ddot{\mathbf{q}}_p(\tau)\} + ([\mathbf{D}_p] + [\mathbf{B}_p(\zeta_a)][\mathbf{g}][\mathbf{C}_p(\zeta_s)] + i\Omega[\mathbf{G}_p])\{\dot{\mathbf{q}}_p(\tau)\} + [\mathbf{K}_p]\{\mathbf{q}_p(\tau)\} = \{\mathbf{N}_p(\tau)\}, \qquad (11)$$

and

$$[\mathbf{M}_r]\{\ddot{\mathbf{q}}_r(\tau)\} + ([\mathbf{D}_r] + [\mathbf{B}_r(\zeta_a)][\mathbf{g}][\mathbf{C}_r(\zeta_s)] + i\Omega[\mathbf{G}_r])\{\dot{\mathbf{q}}_r(\tau)\} + [\mathbf{K}_r]\{\mathbf{q}_r(\tau)\} = \{\mathbf{N}_r(\tau)\}. \qquad (12)$$

where the subscripts p and r refer to the primary and residual mode, respectively. The number of primary and residual modes are n_p and n_r satisfying $n_p + n_r = n_{cr}$. The controller design is based on Eq. (11) of reduced-order model in terms of primary modes, but it is to be implemented on the full order system with residual modes as well. In such case, the constraint in Eq. (10) has to be modified by

$$([\mathbf{D}^*] + [\mathbf{D}_c^*]) > 0, \quad \text{stability criteria} \qquad (13)$$

where

$$[\mathbf{D}_c^*] = \frac{1}{2}\{[\mathbf{B}_f(\zeta_a)][\mathbf{g}][\mathbf{C}_f(\zeta_s)] + ([\mathbf{B}_f(\zeta_a)][\mathbf{g}][\mathbf{C}_f(\zeta_s)])^T\} \qquad (14)$$

represents the control-induced damping matrix for the full-order closed loop system must be semi-definite.

Consider a spinning shaft of slenderness ratio $l = 7$ with linear eccentricity distribution $\varepsilon(\zeta) = \varepsilon_0(1+\zeta)$, and the boundary condition is hinged-hinged. There will be only four critical speeds ($n_{cr} = 4$) at $\Omega_{cr} = 10.13$, 44.18, 120.12 and 358.23 with the corresponding precessional modes $\phi_n(\zeta) = \sin(n\pi\zeta)$, $n = 1, \ldots, 4$. For systems with damping ratio $\xi_1 = \xi_2 = 0.1\%$, the amplitude response $\{\mathbf{q}(\tau))$ are plotted in Fig. 2. The objective of controller design is to place one pair of sensor/actuator in optimal location (ζ_s, ζ_a) with a feedback gain (g) such that the vibration amplitude can be minimized.

If subject to controller order and desired operation range restrictions, only the first two precessional modes is considered in the controller design $n_p = 2$ At the same time, however, the design should prevent the spillover of the remaining modes and guarantee stability. For systems in noncollocated sensor/actuator configuration, the optimal sensor location ζ_s, actuator location ζ_a , and feedback gain (g) are shown in Fig. 3. The first mode is dominant at lower speed, so that the optimal sensor/actuator locations become collocated instead, and they are at the anti-node $\zeta_s = \zeta_a = 0.5$. When the spinning speed $\Omega > \Omega_{cr1}$, the magnitude of q_1 approaches a constant while the influence of q_2 becoming more apparent as shown in Fig. 2. Thus the sensor and actuator locations become noncollocated and they bifurcate into two branches. Their relative distance is determined by the control-induced gyroscopic effect in Eq. (6). The farther apart the sensor and actuator, the more gyroscopic effect induced. But it is also restricted by the stability constraint in Eq. (13). That is why they are kept at a constant distance in order to guarantee stability as shown in Fig. 3.

Figure 4 shows the optimal sensor/actuator locations (ζ_s, ζ_a) and the feedback gain in noncollocated design when all the precessional modes are included $(n_p = 4)$. The unbalanced responses of the optimal design under noncollocated configurations from reduced $(n_p = 2)$ and full order controller $(n_p = 4)$ and are compared in Fig. 5. The reduced-order controller is effective for the first two modes, but it is less effective to for vibration suppression of the residual modes because only the precessional modes within the operation range are considered in the design. Nevertheless, the system remains stable.

四、計畫成果自評

It has been shown that a spinning shaft only has finite number of critical speeds and precessional modes when the whirl ratio $\lambda > 1/2$. Vibration control of steady state unbalanced response by optimal sensor/actuator location and feedback gain are studied analytically in this project. Due to controller order constraint, one can include part of precessional modes in the controller design and then guarantee the spillover stabilization, if any, of the remaining residual modes.

In noncollocated configuration, the optimal sensor and actuator locations are affected not only by the spinning speed but also by the control-induced gyroscopic effect. When operating near one of the critical speeds, the optimal locations become collocated at the anti-node of that critical mode. At the other speeds, the optimal locations are, as expected, noncollocated, and they are kept at a constant distance to ensure system stability. The actuator location is more sensitive than the sensor location to spinning speed and hence to system performance. Under stability constraint, the sensor and actuator locations are placed in a way such that they remain in-phase and the control input is always constructive for vibration suppression.

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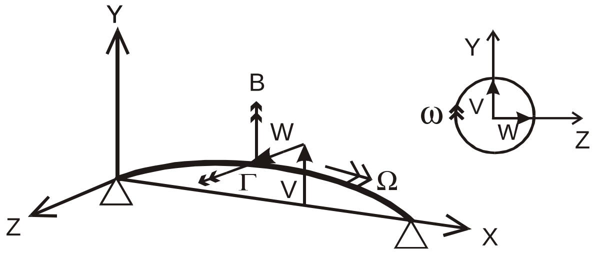

Fig. 1 Schematic diagram of a spinning shaft

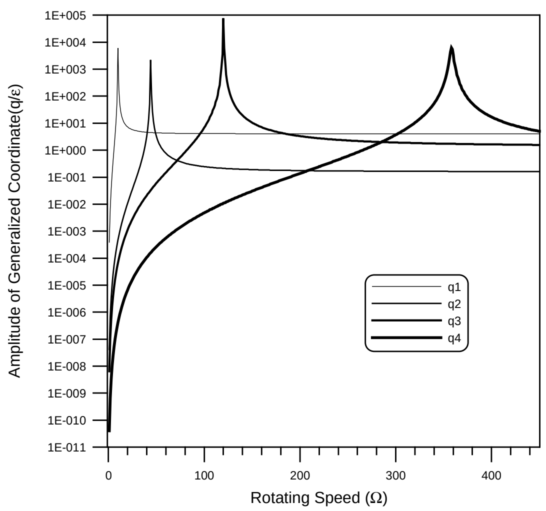

Fig. 2 Amplitude of the generalized unbalanced response $\{\mathbf{q}(\tau)\}$

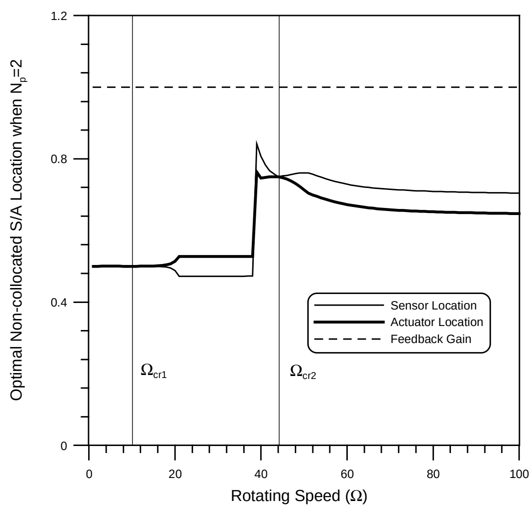

Fig. 4 Optimal sensor/actuator locations (ζ_s, ζ_a) and the feedback gain in noncollocated design when $n_p = 2$.

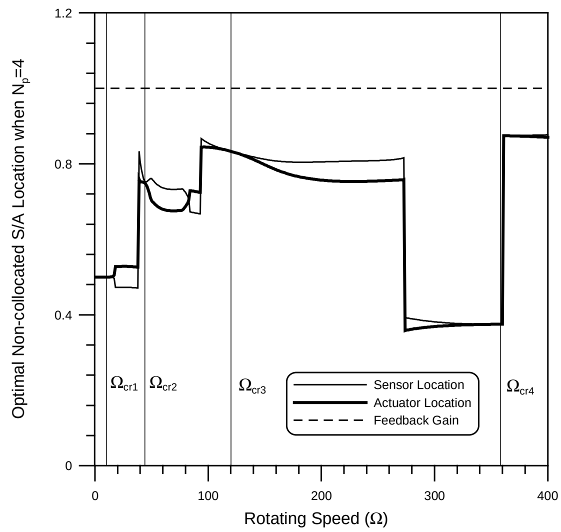

Fig. 5 Optimal sensor/actuator locations (ζ_s, ζ_a) and the feedback gain in noncollocated design when $n_p = 4$.

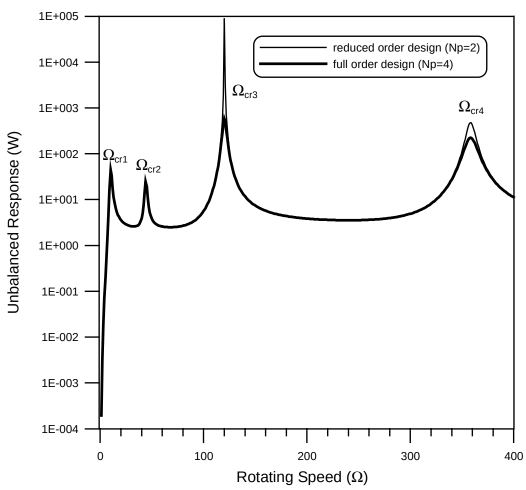

Fig. 6 Unbalanced response of the optimal noncollocated design from reduced $(n_p = 2)$ and full order controller $(n_p = 4)$.

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