行政院國家科學委員會補助專題研究計畫成果報告

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※運用混合數值法求解時態邊界條件下之多層圓柱問題※
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計畫類別：■個別型計畫　　□整合型計畫
計畫編號：NSC　91－2212－E－164－001
執行期間：91 年　1 月 1　日至　91 年 7 月 31 日

計畫主持人：　李宗乙
共同主持人：　陳朝光
計畫參與人員：陳立宏

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執行單位：修平技術學院

中　華　民　國　九十一　年　七　月　三十一　日

運用混合數值法求解時態邊界條件下之多層圓柱問題

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一、中文摘要

本文主要是分析一多層不同材料所組成的空心圓柱，因壁內流體加溫過程中溫度與壓力變化下而所產生的熱應力問題。而我們將使用有限差分法與拉氏轉換來處理此類問題。利用拉氏轉換法處理時間項，再利用矩陣相似轉換在轉換域上求得簡單的複變函數解。最後利用部分分式法解簡單的數值逆拉氏轉換，求得數值解。

關鍵詞：有限差分、拉氏轉換、相似轉換

Abstract

This paper deals with one-dimensional axisymmetric quasi-static coupled thermoelastic problems with time-dependent boundary conditions. Laplace transform and finite difference methods are used to analyze the problems. Using the Laplace transform with respect to time, the general solutions of the governing equations are obtained in transform domain. The solution is obtained by using the matrix similarity transformation and inverse Laplace transform. We obtain solutions for the temperature and thermal deformation distributions for a transient and steady state.

Keywords: Laplace transform, finite difference methods, matrix similarity transformation

二、INTRODUCTION

There have been a lot of papers dealing with thermoelasticity problems. Takeuti and Furukawa [1] discussed the thermal shock problems in a plate, they include the inertia and thermoelastic coupling terms in the governing equation and obtained the exact solution for thermal shock problem in the plate. Sherief and Anwar [2] discussed the problem of an annular infinitely long elastic circular cylinder whose inner and outer surfaces are subject to known temperature and are traction free. Yang and Chen [3] discussed the transient response of one-dimensional quasi-static coupled thermoelasticity problems of an infinitely long annular cylinder composed of two different materials.

Chen et al. [4-7] presented a new numerical technique—hybrid numerical method for the problem of a transient linear heat conduction system. He applied the Laplace transform to remove the time-dependence from the governing equation and boundary conditions, and solved the transformed equations with the finite element and finite difference method. Finally the transformed temperature was inverted by numerical inversion of the Laplace transform. It proved that the method can accurately determine the stable solutions at a specific time.

The present work deals with one-dimensional quasi-static coupled thermoelastic problems of an infinitely long multilayered hollow cylinder composed of multilayered different ceramic-metal materials and boundary conditions.

三、Formulation

The layered cylindrical shell to be analyed is shown in Fig. 1. The transient heat conduction equation for the ith layer in dimensional form can be written as

$$[k_r \frac{\partial^2}{\partial r^{*2}} + k_\theta \frac{1}{r^*}\frac{\partial}{\partial r^*}]\overline{\Theta} = \rho C_v \frac{\partial \overline{\Theta}}{\partial \tau} + \Theta_0 \beta_r \frac{\partial}{\partial r^*}(\frac{\partial U}{\partial \tau}) + \Theta_0 \beta_\theta \frac{1}{r^*}(\frac{\partial U}{\partial \tau}) \tag{1}$$

Where $\overline{\Theta} = \Theta - \Theta_0$,

$\beta_r = \frac{E_r}{1-\nu_{r\theta}\nu_{\theta r}}(\alpha_r + \nu_{\theta r}\alpha_\theta)$ and

$\beta_\theta = \frac{E_\theta}{1-\nu_{r\theta}\nu_{\theta r}}(\alpha_\theta + \nu_{r\theta}\alpha_r)$

If the body forces are absent, the equation of equilibrium for a cylinder along the radial direction can be written as

$$\frac{\partial^2 U}{\partial r^{*2}} + [\frac{E_\theta}{E_r}\nu_{\theta r} + (1-\nu_{r\theta})]\frac{1}{r^*}\frac{\partial U}{\partial r^*} - \frac{E_\theta}{E_r}\frac{1}{r^{*2}}U = (\alpha_r + \nu_{\theta r}\alpha_\theta)\frac{\partial \overline{\Theta}}{\partial r^*} - [\frac{E_\theta}{E_r}(\alpha_\theta + \nu_{r\theta}\alpha_r) - (\alpha_r + \nu_{\theta r}\alpha_\theta)]\frac{\overline{\Theta}}{r^*} \tag{2}$$

The stress-displacement relations are

$$\sigma^*_{ri} = (\frac{E_r}{\lambda})_i \frac{\partial U}{\partial r^*} + (\frac{E_\theta \nu_{\theta r}}{\lambda})_i \frac{U}{r^*} - \beta_{ri}(\Theta - \Theta_0) \tag{3}$$

$$\sigma^*_{\theta i} = (\frac{E_r \nu_{r\theta}}{\lambda})_i \frac{\partial U}{\partial r^*} + (\frac{E_\theta}{\lambda})_i \frac{U}{r^*} - \beta_{\theta i}(\Theta - \Theta_0) \tag{4}$$

Let the boundary surfaces of composite cylinder be traction free and subjected to time-dependent or constant temperatures. The initial and boundary conditions are

$U = \dot{U} = \overline{\Theta} = \dot{\overline{\Theta}} = 0$ at $t = 0$

$\sigma^*_r(r,t) = 0$, $\Theta_1 - \Theta_0 = f_1$ at $r^* = R_1$

Case 1

$\sigma^*_r(r,t) = 0$, $\frac{\partial \Theta}{\partial r^*} = 0$ at $r^* = R_{out}$

Case 2

$\sigma^*_r(r,t) = 0$, $-k\frac{\partial \Theta}{\partial r^*} = h(\Theta - \Theta_\infty)$ at $r^* = R_{out}$

At the interface between two adjacent layers, the following matching conditions must be satisfied:

$U_i(r,t) = U_{i+1}(r,t)$, $\sigma^*_{ri}(r,t) = \sigma_{ri+1}(r,t)$

$q_i = q_{i+1}$, $\overline{\Theta}_i(r,t) = \overline{\Theta}_{i+1}(r,t)$

The governing equations and stress-displacement relations have the nondimensional form. Taking the Laplace transform and central difference for equations, we obtain the following equation in matrix

$$\{[M] - s[I]\}\{\overline{T}_j\} + s[N]\{\overline{u}_j\} = \{\overline{G}_j\} \tag{5}$$

$$[R]\{\overline{T}_j\} + [Q]\{\overline{u}_j\} = \{\overline{V}\} \tag{6}$$

Substituting equation (5) into (6), we have

$$\{[A] - s[I]\}\{\overline{T}_j\} = \{\overline{F}_j\} \tag{7}$$

Since the ($N \times N$) matrix [A] is a nonsingular real matrix, the matrix [A] possesses a set of N linearly independent eigenvectors, hence the matrix [A] is diagonalizable. Equation can be rewritten as

$$\{\text{ diag[A] - } s[I]\}\{\overline{T}^*_j\} = \{\overline{F}^*_j\} \tag{8}$$

where $\{\overline{T}^*_j\} = [P]^{-1}\{\overline{T}_j\}$, $\{\overline{F}^*_j\} = [P]^{-1}\{\overline{F}_j\}$

From equation (8), the following solutions can be obtained immediately.

$$\overline{T}^*_j = \frac{\overline{F}^*_j}{\lambda_j - s} \qquad j = 1,2, \ldots, N \tag{9}$$

By applying the inverse Laplace transform to equation (9), we get the solution T^*_j. After we have obtained T^*_j, then we can use following equations (10) and (11) to obtain the solutions T_j and u_j

$$\{T_j\} = [P]\{T^*_j\} \tag{10}$$

$$\{u_j\} = [Q]^{-1}[\overline{V}] - [Q]^{-1}[R]\{T_j\} \tag{11}$$

Substituting T_j and u_j into equations, we obtain the radial and circumferential stresses.

Numerical Result and Discussions

In this study, we present some numerical results for the temperature distributions in a long multilayered composite hollow cylinder, subjected to the considered boundary conditions and the resulting displacement and thermal stresses (Fig 2-12).

For an infinitely long annular multilayered cylinder, the geometry and material quantities of the cylinder are shown in Table 1. The inner and outer radius of the cylinder are assumed to be 1.0 and 4.5 respectively. The case 1 boundary conditions at inner and outer surfaces are assumed to be *f(t)* and adiabatic respectively. The case 2 boundary conditions at inner and outer surfaces are assumed to be *f(t)* and convective respectively. Each layer is assumed to have a different thickness (in the case of three layers, $\Delta h_1 = 1.$, $\Delta h_2 = 1.$ and $\Delta h_3 = 1.5$).

References

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	layer 1 Titanium	layer 2 Al_2O_3	layer 3 Steel(1025)
$E_r = E_\theta$	108E9	390E9	207E9
$k_r = k_\theta$	20	6	17
$\nu_{r\theta} = \nu_{\theta r}$	0.3	0.23	0.3
$\alpha_r = \alpha_\theta$	11E-6	8E-6	11E-6
ρ	4	3.99	7.8
C_v	0.4	1.25	0.48

Table 1. The geometry and material constants of an infinitely long multilayered cylinder ($R_{out}/R_1 = 4.5$, $h = 200$, $\Theta_0 = T_\infty = 298\ K$)

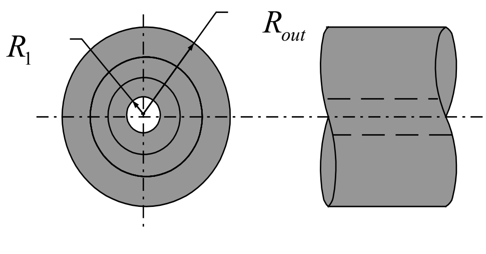


Fig. 1. Physical model and system coordinates Multilayered infinitely long cylinder

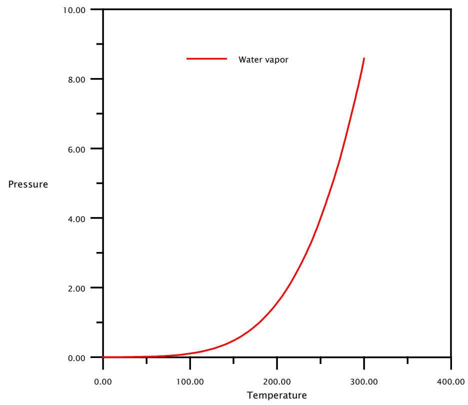
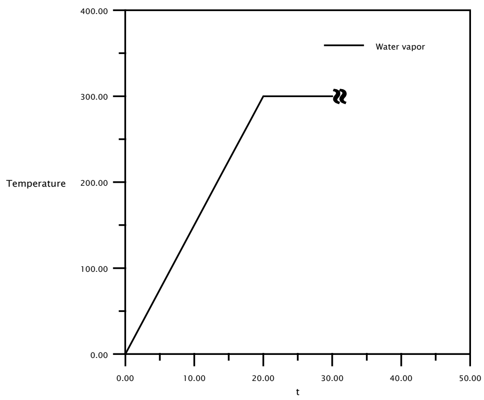
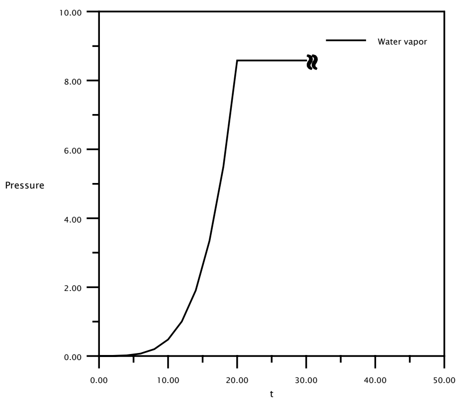


Fig. 2. Temperature and pressure relation in inner boundary (quality 90%)



$$f(t)=\begin{cases}15t & , \quad 0\le t\le 20\\ 300 & , \quad t>20\end{cases}$$

Fig. 3. Temperature distribution with time in inner boundary



$$p(t)=\begin{cases}-0.00013995t+0.00461523t^2 & \\ -0.00112473t^3+0.00060406t^4 & ,0\le t\le 20\\ 8.585 & ,t>20\end{cases}$$

Fig. 4. Pressure distribution with time in inner boundary

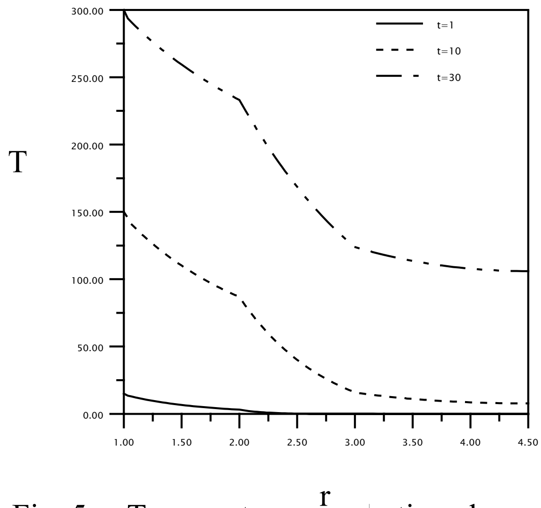


Fig. 5. Temperature distribution along radial direction for adiabatic case

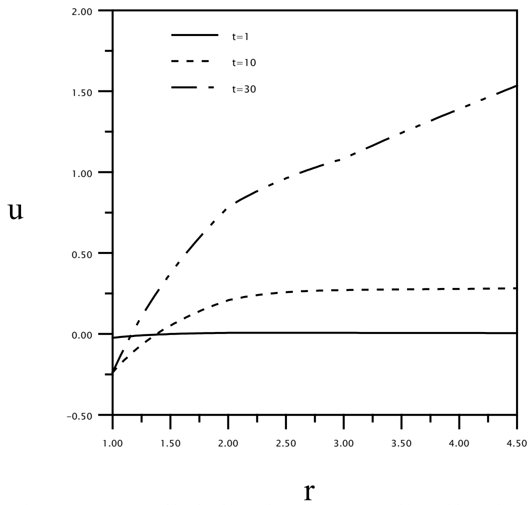


Fig. 6. Radial displacement distribution along radial direction for adiabatic case

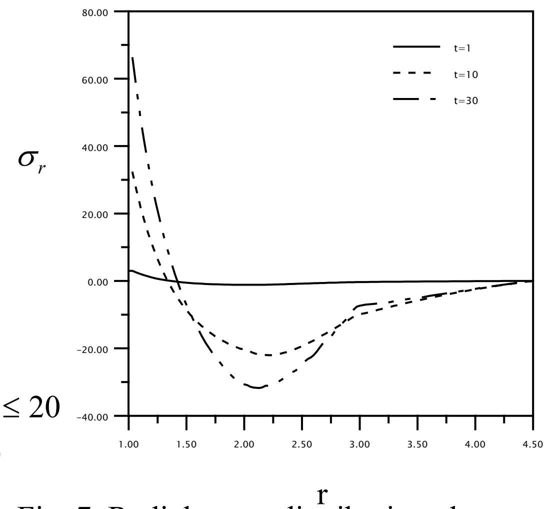


Fig. 7. Radial stress distribution along radial direction for adiabatic case

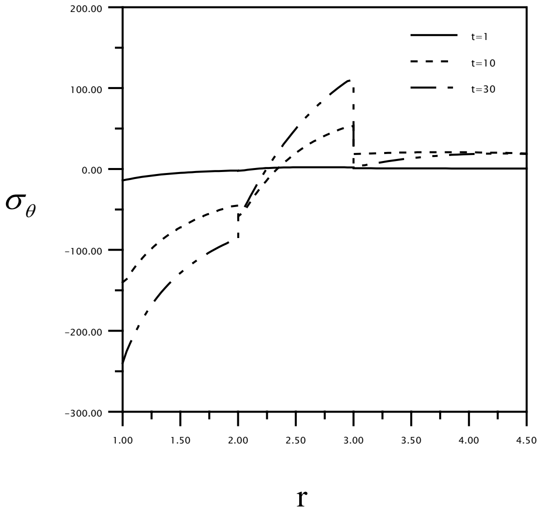


Fig. 8. Circumferential stress along radial direction for adiabatic case

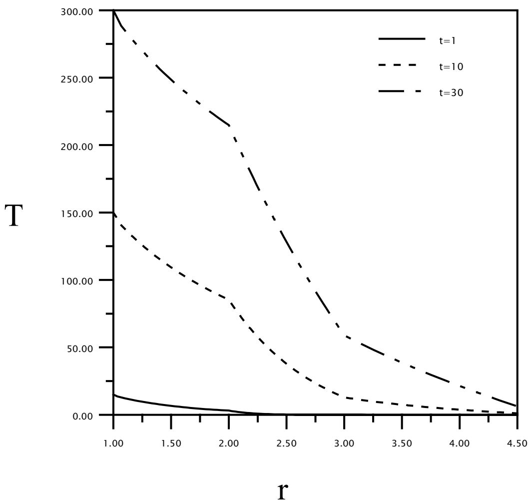


Fig. 9. Temperature distribution along radial direction for convective case

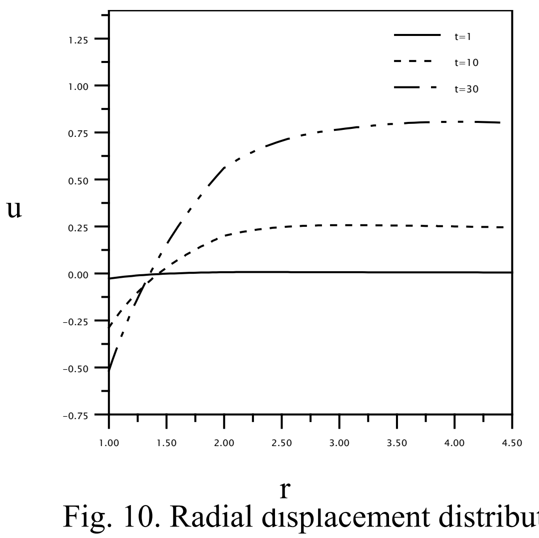


Fig. 10. Radial displacement distribution along radial direction for convective case

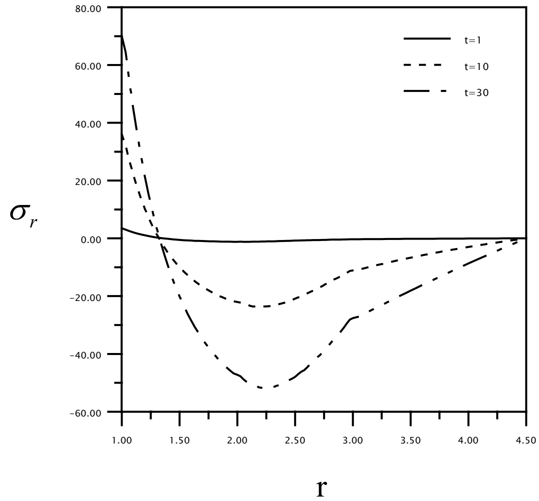


Fig. 11. Radial stress distribution along radial direction for convective case

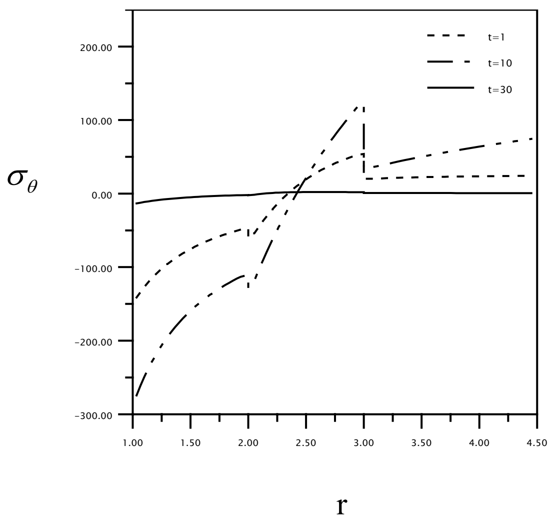


Fig. 12. Circumferential stress along radial direction for convective case