The Design of sliding-and-classical controllers for Improper System

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Abstract

This paper is concerned with the development of a method for designing a sliding mode and classical controller. The idea is based on defining the sliding variable in such a way that once the system gets into sliding, the classical controller transfer function can be realized, when realizing an improper controller. The advantage of this sliding-and-classical controller is that it can retain all the merits of both types controllers on one and eliminates their respective limitations on the other. For the improvement of the existing design method, the number of differentiation of the sliding surface is reduced. The proposed method is robust and applies to non-minimum phase systems as well as systems with structural uncertainties. No state measurement is required.

Key word: Sliding mode controller、non-minimum phase.

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Ⅰ. Introduction

These traditional continuous-data controllers are the most popular controllers for industry, and have been widely used in many applications [1-3]. They are simple, easy to design and mature in theoretical background. But there are also limitations of these controllers. One major limitation is that there should be low derivative actions. This is necessary in order to avoid noise enhancement.

Sliding mode control has been studied for years, and has been successfully used in many applications [4-7]. Yeung *et. al.*[8] has provided a novel designing method for the sliding mode controller, which combines the design technique of the classical controller and traditional sliding mode controller. This hybrid design method not only incorporates the advantages of the sliding mode controllers and the classical controllers, but also eliminates their limitations. But some disadvantages are also found in this paper:

1. The sliding surface variable must have m_c differentiators.
2. It lacks of theoretical analysis when system contains noise.

The object of this paper is to reduce the number of differentiators of sliding variable. So that when an improper controller is implemented, the system will have strong derivative actions but without noise enhancement.

Ⅱ. Mathematical Model

Consider a linear time-invariant Single-Input-Single-Output plant described by the output equation

$$y = G(p)u + d \tag{2.1}$$

$$G(p) = \text{plant transfer function } = \frac{N(p)}{D(p)}$$

$$\deg N(p) = m,\ \deg D(p) = n,\ m \le n$$

where p is the differential operator and d represents a large of possible signals. In order to widen the application of the controllers, the disturbances, noise and model uncertainties should be taken considered.

Consider a stable classical control system (reference fig.1)

$$\frac{u(p)}{e(p)} \equiv KG_c(p) = K\frac{N_c(p)}{D_c(p)} \tag{2.2}$$

In order to improve the designing method in [8] and to realize the linear improper controller (2.2) by the sliding mode technique, the continuous scalar sliding variable can be chosen as

$$\sigma = \frac{N_c(p)}{D_c(p)}e - \frac{1}{K}u = G_c(p)e - \frac{1}{K}u \tag{2.3}$$

where

$G_c(p) = G_{cd}(p) + G_{ci}(p)$

$G_{ci}(p) = N_{ci}(p)/D_c(p)$ is a strictly proper transfer function.

In order to reduce the number of differentiator in (2.3) further, we can replace p by $(p/\varepsilon p+1)$ in $G_{cd}(p)$ to obtain another transfer function $G_{ca}(p)$, where ε is a positive number, i.e.,

$$G_{ca}(p) = \frac{N_{cda}(p)}{D_{cda}(p)} + \frac{N_{ci}(p)}{D_c(p)} \tag{2.4}$$

$$\equiv G_{cda}(p) + G_{ci}(p) \tag{2.5}$$

where $N_{cda}(p) = p^{m_c-n_c} + \gamma_{m_c-n_c-1}p^{m_c-n_c-1}(\varepsilon p+1)+\cdots+\gamma_0(\varepsilon p+1)^{m_c-n_c}$ (2.6)

$$D_{cda}(p) = (\varepsilon p+1)^{m_c-n_c} \tag{2.7}$$

In summary, if $m_c \le n_c$, the sliding variable is chosen as

$$\sigma = G_c(p)e - \frac{1}{k}u$$

i.e. $N_{cda}(p) = 0$, $D_{cda}(p) = 1$, $N_{ci}(p) = N_c(p)$

but if $m_c > n_c$, then the sliding variable is chosen as

$$\sigma = \left(\frac{N_{cda}(p)}{D_{cda}(p)} + \frac{N_{ci}(p)}{D_c(p)} \right) e - \frac{1}{K}u = G_{ca}(p)e - \frac{1}{K}u \tag{2.8}$$

Ⅲ. Controller Design

After designing the sliding variable, the next step is to design a control effort u to drive the system into sliding. (reference fig.2)

(a) Introduction a nominal plant

A nominal plant model $G_0(p)$ with relative degree $\delta_0 = \delta_\ell$ is used to generate a nominal output response according to

$$\hat{y} = G_0(p)u \tag{3.1}$$

Then we obtain

$$\dot{\sigma} = pG_{ca}(p)(y_d - \hat{y}) + pG_{ca}(p)\tilde{e} - \frac{p}{K}u \tag{3.2}$$

(b)Introduction of a filter

In order to generate a low-noise approximation signal v for $pG_c(p)\tilde{e}$, an all-pole filter of transfer function $1/W(p)$ is used to yield

$$v = \frac{pG_{ca}(p)}{W(p)}\tilde{e} = \frac{pG_{cda}(p)}{W(p)}\tilde{e} + \frac{pG_{ci}(p)}{W(p)}\tilde{e} \tag{3.3}$$

Using the filter in (3.3), the second term in (3.2) becomes

$$pG_{ca}(p)\tilde{e} = W(p)v = v + \rho \tag{3.4}$$

Substituting (3.4) into (3.2) yields

$$\dot{\sigma} = pG_{ca}(p)(y_d - \hat{y}) + v + \rho - \frac{p}{K}u \tag{3.5}$$

(c) Introduction of an auxiliary control

This is done by introducing an auxiliary control

$$\tilde{u} = \frac{p}{K}u \tag{3.6}$$

so that (3.5) becomes

$$\dot{\sigma} = pG_{ca}(p)(y_d - \hat{y}) + v + \rho - \tilde{u} \tag{3.7}$$

Now let

$$\tilde{u} \equiv \tilde{u}_c + \tilde{u}_s \tag{3.8a}$$

$$= pG_{ca}(p)(y_d - \hat{y}) + v + K_\sigma sat(\frac{\sigma}{K_s}) \tag{3.8b}$$

where

$$\tilde{u}_s = K_\sigma sat(\frac{\sigma}{K_s}) \tag{3.8c}$$

$$sat(\frac{\sigma}{K_s}) \equiv \begin{cases} 1 & for \quad \sigma \geq K_s \\ \frac{\sigma}{K_s} & for \quad -K_s < \sigma < K_s \\ -1 & for \quad \sigma \leq K_s \end{cases} \tag{3.8d}$$

is the switching part of the auxiliary control, K_σ and K_s are positive constants.

Substituting (3.8a) into (3.7) yields

$$\dot{\sigma} = -K_\sigma sat(\frac{\sigma}{K_s}) + \rho \tag{3.9}$$

It's very easy to verify that the sliding condition $\sigma\dot{\sigma} < 0$ is satisfied in the regions $|\sigma| \geq K_s$ if

$$K_\sigma > |\rho| \tag{3.10}$$

IV. Computer Simulation

In this section we demonstrate an example in which the sliding mode control technique is used to implement a linear improper continuous-data controller [9]. We design a sliding mode controller with the theoretical background based on the previous sections. The results are simulated by using the software package SIMNON-for Windows.

The input-output frequency-response-matching method is used for modeling the system. Its low-frequency linear approximate transfer function is obtained as

$$G(s)=\frac{250s+47500}{s^3+230.01s^2+6002.3s+60} \tag{4.1}$$

and the nominal plant is assumed to be

$$G_0(s)=\frac{250s+45000}{s^3+230s^2+6002s+60}. \tag{4.2}$$

The classical improper controller that we want to implement is

$$G_c(s)=\frac{1.5\times(s+20)\times(s+100)}{s+50} \tag{4.3}$$

When d(t) is the disturbance, it is assumed to be

$$d(t)=0.15\times(t-0.01)^2\times exp\left(-\frac{(t-0.01)^2}{2}\right)\times u(t-0.01) \tag{4.4}$$

The reference input is the unit-step function. The parameters of the controller are chosen as follows:

$$K_\sigma=1.5\times10^3,\ K_s=10^{-2} \tag{4.5}$$

and the parameter of differentiator $\varepsilon=5\times10^{-4}$

The filter with second order polynomial

$$W(s)=s^2+2s+1 \tag{4.6}$$

is also used in this case. The simulation result is shown in Fig.3.

V. Conclusions

This paper introduces a sliding-and-classical controller by using sliding mode control technique. Not only a traditional improper controller can be realized, but also is the noise enhancement problem avoided. Through the simulation of computer, the sliding-and-classical controlling method can realize the improper controller and avoid the effects of the disturbance problem.

VI.Reference

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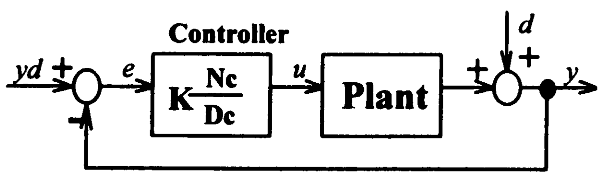

Fig.1 A stable classical control system

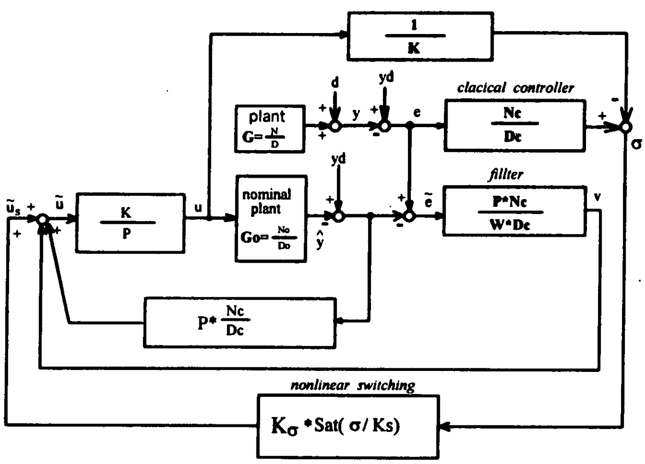

Fig.2 A sliding-mode control system

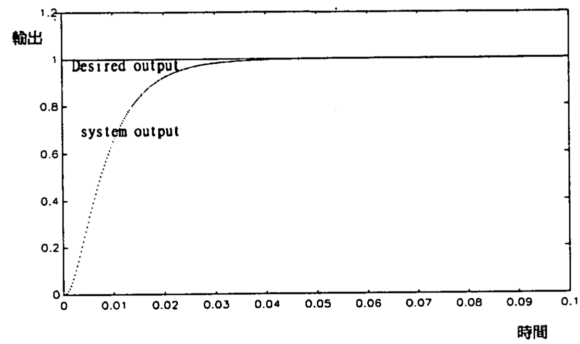

Fig.3 Simulation result

假分式系統之順滑與古典控制器設計

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摘　要

本文是要發展一套順滑與古典控制器 (sliding mode and classical controller) 設計法。其構想是定義一順滑參數 (sliding variable)，使系統進入順滑時，可使假分式 (improper) 之古典控制器的特性可顯現出來。此種順滑與古典控制器的優點在於它能保持順滑模態控制器與古典控制器的優點，而也能去除兩者之缺點。在改進設計方法方面，我們減少了順滑面微分的次數，此新方法具有強健性並且能應用到非極小相位系統，而系統之狀態也不需量測　。

關鍵詞：順滑控制器、非極小相位

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