Game Theoretic Approach in the 3D Nonlinear H_∞-based Guidance Design

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Abstract

In this paper, its features are: 1.The complete nonlinear dynamics of the pursuit-evasion motion is considered in three-dimensional spherical coordinate system. Neither linearization nor small signal assumptions are made. 2.The nonlinear H_∞ guidance design is derived analytically and expressed in a very simple form. 3.Unlike adaptive control concept, implementation of the proposed H_∞ guidance design does not need the information on target acceleration while ensuring acceptable intercept performance for arbitrary targets with the finite acceleration. 4.The derived guidance design exhibits strong robustness against variations in target acceleration.

Key Words: H_∞ guidance design, game theory, spherical coordinate system

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利用對局理論逼近法設計三維度非線性H_∞飛彈導引律

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摘要

本文的目的主要是直接採用非線性H_∞理論來處理飛彈的飛行控制問題。利用H_∞的理論架構，配合飛彈六個自由度的飛行動態方程式，吾人可將飛彈的橫向與縱向的飛行控制問題轉化成非線性H_∞的干擾濾除問題(nonlinear H_∞ disturbance rejection problem)。此外吾人根據耗散性原理，透過求解一組非線性Hamilton-Jacobi微分不等式(nonlinear Hamilton-Jacobi differential inequality (HJPDI))，可求解出一組飛彈控制律，且此解不僅滿足H_∞之性能要求，而且其型式僅爲簡單之比例回授控制律。

關鍵字：H_∞ guidance design, game theory, spherical coordinate system

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Introduction

This paper presents the direct use of nonlinear H_∞ control theory to the missile complete six degree-of-freedom nonlinear flight dynamics, which has not been considered in the literature before. Under this approach, the exact nonlinear equations governing both the longitudinal and lateral motions are considered, and the missile control design is formulated as a nonlinear H_∞ disturbance rejection problem where unmodeled aerodynamics, and unpredictable disturbances (maneuvering targets). The characterization of the nonlinear H_∞ control law relies on the solution of a first-order, second-degree nonlinear Hamilton-Jacobi differential inequality (HJPDI). An excellent analytical solution for missile control application is found in this paper by a simple form which is applied by the dissipative theory, and the resulting nonlinear H_∞ control law is shown to be in a simple structure of proportional feedback.

A robust control approach for future missile autopilot design was present in Bbuschek (1997), Hyde (1995), and Ferreres et al. (1994). Wise (1997) has proposed a nonlinear H_∞ approach for high AOA missile agile missile by approximating the solution to the HJPDI equation and a state dependent Riccati Equation (SDRE) developed to solve for a nonlinear H_∞ control that satisfied the HJPDI. This methodology recently developed by Yang and Chen (1998) for nonlinear H_∞ guidance design synthesis. This approach is systematic and valid for general vehicle. A remarkable property of the derived nonlinear H_∞ missile controller is that the desired control force and control moment to reject the disturbances can be computed quantitatively in advance without knowing the information of flight vehicle's aero data. After the control force and moment have been found, the required control surface deflections can then be determined from the look-up aero tables of the flight vehicle to be controlled. This property is unlike the conventional control design where aerodynamic model must be established before any control activity can be made. Due to the aerodynamic irrelevance in constructing the nonlinear H_∞ missile controllers, the derived control law can be equally applied to airplane by Kung（2000）, helicopters, and other Right vehicles.

Dissipation and Nonlinear H Control

In this section, we apply the dissipative theory to derive the standard results of the nonlinear H_∞ control theory for later use. The dissipative theory generalizes the idea of passivity, and provides a means of robust stabilization for nonlinear systems. Consider a general nonlinear system in the form

$$\dot{x} = f(x(t)) + g(x(t))w, f(0) = 0 \quad (1a)$$

$$z(t) = h(x(t), w(t)), h(0) = 0 \quad (1b)$$

Where $w(t)$ is the input vector, and $z(t)$ is the penalized output vector. Associated with this nonlinear system, there is a quadratic function $r(t)$ which is called supply rate. The supply rate $r(t)$ is selected based on the system properties, for example whether it is norm bounded or passive. System (1) is said to be dissipative with respect to the supply rate $r(\cdot,\cdot)$, if there exists an energy storage function $V(x)$, $\forall x \neq 0$ satisfying the following dissipative inequality:

$$0 \le V(x(t)) \le \int_0^t r(w(\xi), z(\xi))d\xi \quad (2)$$

for all t and for all $x(\bullet)$, $z(\bullet)$, and $w(\bullet)$ satisfying Eqs. (1). If $V(x)$ is a differentiable function, then an equivalent statement of dissipativity (2) is

$$\frac{dV(x(t))}{dt} \le r(w(t), z(t)), \forall t \ge 0 \quad (3)$$

where

$$dV(x(t))/dt = \frac{\partial V}{\partial x}\dot{x} = \left[\frac{\partial V}{\partial x_1}, \frac{\partial V}{\partial x_2}, \ldots \frac{\partial V}{\partial x_n}\right]\dot{x}$$

denotes the total derivative of $V(x^\circ(t))$ along the state trajectory $x(t)$. The supply rate r related to H_∞ control problem is defined as

$$r(w(t), z(t)) = \gamma^2 w^T w - z^T z \quad (4)$$

Now, we assume that f (x), h (x) are C^∞ functions （C^∞ belong to the complex space） and x=0 is the equilibrium point of the system, i.e., f (0) = h (0) = 0.

If there exists a scalar C^1 function $V: R^n \rightarrow R^+$ with V (0) = 0 such that

$$\frac{1}{2\gamma^2}\left(\frac{\partial V}{\partial x}\right)^T g(x)g(x)^T\left(\frac{\partial V}{\partial x}\right)+\left(\frac{\partial V}{\partial x}\right)^T f(x)+\frac{1}{2}h^T(x)h(x)<0 \quad (5)$$

, then the system is said to have L_2 gain < γ, i.e.

$$\int_0^T (z^T z)dt < \gamma^2 \int_0^T (w^T w)dt \quad (6)$$

Since $\int_0^T w^T w dt$ and $\int_0^T z^T z dt$ are the input energy and the output energy of the system, respectively, the system satisfying Eq. (6), according to the definition, then has L_2-gain lower than or equal to γ. The smallest γ satisfying Eq. (6) is called the H_∞-norm of the system. The equivalence between Eq. (4) and Eq. (6) reveals that a system which has L_2-gain γ is a dissipative system, and vice versa. Hence, from the viewpoint of dissipation, we can say that the nonlinear H_∞ control technique is a means to make a nonlinear system dissipative.

When control u. is applied to the system, we obtain the controlled system as:

$$\dot{x} = f(x) + g_1(x)w + g_2(x)u \quad (7a)$$

$$z = \begin{bmatrix} h_1(x) \\ \rho_u u \end{bmatrix} \quad (7b)$$

where ρ_u, is a weighting coefficient. The nonlinear H_∞, control problem is to find the control u such that the L_2-gain of the closed-loop system is less than γ. By replacing f (x), g (x), and h (x) in Eq. (2) with $f(x)+g_2(x)u, g_1(x)$, and $[h_1^T(x)\rho_u u^T]^T$, respectively, the condition that the L_2-gain of the closed-loop system is lower than γ, becomes

$$\left(\frac{\partial V}{\partial x}\right)^T f+\frac{1}{2}\left(\frac{\partial V}{\partial x}\right)^T\left(\frac{1}{\gamma^2}g_1 g_1^T-\frac{1}{\rho_u^2}g_2 g_2^T\right)\left(\frac{\partial V}{\partial x}\right)+\frac{1}{2}h_1^T h_1<0 \quad (8)$$

and control law u

$$u(x) = -\frac{1}{\rho_u^2} g_2^T(x)\left(\frac{\partial V}{\partial x}\right) \tag{9}$$

Hence, solving the nonlinear H_∞, control problem is equivalent to finding a positive function V (x) satisfying HJPDI. The corresponding HJPDI for the flight control problem will be derived in the next section. It is worth noting that the existence of a positive V (x) satisfying HJPDI guarantees the boundedness of the state z (x) in the sense of Lyapunov stability.

In the following flight control formulation, the manipulations of cross-product matrix will be frequently referred to, and the cross-product matrix S (K) induced by the vector K=[k_1 k_2 k_3]^{T}defined as

$$S(K) = \begin{bmatrix} 0 & -k_3 & k_2 \\ k_3 & 0 & -k_1 \\ -k_2 & k_1 & 0 \end{bmatrix} \tag{10}$$

Missile Dynamical Model and Aerodynamics

The basic set of axes used throughout this paper is defined in Fig.1. Axes shown here is a set of body axes, x, y, and z fixed the missile. Note that the origin of this rotating coordinate is the center of gravity. The resultant of external forces (aerodynamic force and thrust) acting on the missile can be decomposed into three components along x, y, z-axis, which are X, Y, and Z, respectively. The six degree-of-freedom rigid body motion of a flight vehicle can be described by the following differential equations:

$$m_s \dot{U} = m_s(-WQ + VR) + F_x + w_x \tag{11a}$$

$$m_s \dot{V} = m_s(-UR + WP) + F_y + w_y \tag{11b}$$

$$m_s \dot{W} = m_s(-VP + UQ) + F_z + w_z \tag{11c}$$

$$I_{xx}\dot{P} = -I_{xz}(\dot{R} + PQ) - I_{xy}(\dot{Q} - PR) + I_{yz}(R^2 - Q^2) + (I_{yy} - I_{zz})QR + L + w_l \tag{11d}$$

$$I_{yy}\dot{Q} = -I_{xy}(\dot{P} + QR) - I_{yz}(\dot{R} - PQ) + I_{xz}(P^2 - R^2) + (I_{zz} - I_{xx})PR + M + w_m \tag{11e}$$

$$I_{zz}\dot{P} = -I_{yz}(\dot{Q} + PR) - I_{xz}(\dot{R} - QR) + I_{xy}(Q^2 - P^2) + (I_{xx} - I_{yy})PQ + N + w_n \tag{11f}$$

where U, V, W, and P, Q, R are standard notations for linear and angular velocities, respectively; I_{xx}, I_{xz},⋯, etc, are the moments of inertia of the flight vehicle; m, is the vehicle's mass. F_x, F_y, F_z and L, M, N are the applied forces and moments which are accessible from the models of gravity, aerodynamics, and thrust, while w_x, w_y, w_z, and w_l, w_m, w_n are the applied forces and moments resulting from the unmodeled aerodynamics or from the unpredictable disturbance such as maneuvering targets. Eqs. (11) can be reformulated to a compact matrix form which is more suitable for nonlinear control design.

$$\begin{bmatrix}\dot{U}\\ \dot{V}\\ \dot{W}\end{bmatrix}=-\begin{bmatrix}0 & -R & Q\\ R & 0 & -p\\ -Q & P & 0\end{bmatrix}\begin{bmatrix}U\\ V\\ W\end{bmatrix}+\frac{1}{m_s}\begin{bmatrix}F_x\\ F_y\\ F_z\end{bmatrix}+\frac{1}{m_s}\begin{bmatrix}w_x\\ w_y\\ w_z\end{bmatrix} \tag{12a}$$

$$\begin{bmatrix}I_{xx} & I_{xy} & I_{xz}\\ I_{xy} & I_{yy} & I_{yz}\\ I_{xz} & I_{yz} & I_{zz}\end{bmatrix}\begin{bmatrix}\dot{P}\\ \dot{Q}\\ \dot{R}\end{bmatrix}=-\begin{bmatrix}0 & -R & Q\\ R & 0 & -P\\ -Q & P & 0\end{bmatrix}\begin{bmatrix}I_{xx} & I_{xy} & I_{xz}\\ I_{xy} & I_{yy} & I_{yz}\\ I_{xz} & I_{yz} & I_{zz}\end{bmatrix}\begin{bmatrix}P\\ Q\\ R\end{bmatrix}+\begin{bmatrix}L\\ M\\ N\end{bmatrix}+\begin{bmatrix}w_l\\ w_m\\ w_n\end{bmatrix} \tag{12b}$$

To further simplify the notations, the following definitions are used throughout the paper.

$$\begin{aligned}
&\Sigma(t)=[U \quad V \quad W]^T=[U_0 \quad V_0 \quad W_0]^T+[u \quad v \quad w]^T=\Sigma_0+\sigma(t)\\
&\Omega(0)=[P \quad Q \quad R]^T=[P_0 \quad Q_0 \quad R_0]^T+[p \quad q \quad r]^T=\Omega_0+\omega(t)\\
&u_\Sigma=[F_x \quad F_y \quad F_z]^T=[F_{x0} \quad F_{y0} \quad F_{z0}]^T+[f_x \quad f_y \quad f_z]^T=u_{\Sigma_.}+u_\sigma\\
&u_\Omega=[L \quad M \quad N]^T=[L_0 \quad M_0 \quad N_0]^T+[l \quad m \quad n]^T=u_{\Omega_.}+u_\omega\\
&w_\sigma=[w_x \quad w_y \quad w_z]^T, w_\omega=[w_l \quad w_m \quad w_n]^T
\end{aligned}$$

where the symbols with subscript zero denote the values at equilibrium point (trim condition), and the lower-case symbols denote the deviation from the equilibrium point. However, it needs to be noted here that we do not make the assumption of small deviation, i.e., the nonlinear terms $\sigma^T\sigma$ and $\omega^T\omega$ are not be negligible when compared with the linear terms σ and ω. In terms of the notations defined above, Eqs. (12) can be recast into the following form:

$$\dot{\sigma} = -S(\Omega_0 + \omega)(\Sigma_0 + \sigma) + \frac{1}{m_s}(u\Sigma_0 + u_\sigma) + \frac{1}{m_s} w_\sigma \quad (13a)$$

$$\dot{w} = -I_M^{-1} S(\Omega_0 + \omega) I_M (\Omega_0 + \omega) + I_M^{-1}(u\Omega_0 + u_w) + I_M^{-1} w_\omega \quad (13b)$$

where I_M is the matrix formed by the moments of inertia of the flight vehicle:

$$I_M = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

The cross-product matrix S(·) has been defined in Eq.

$$u\Sigma_0 = \begin{bmatrix} F_{x0} \\ F_{y0} \\ F_{z0} \end{bmatrix} = m_s S(\Omega_0)\Sigma_0 = m_s \begin{bmatrix} -R_0 V_0 + Q_0 W_0 \\ R_0 U_0 - P_0 W_0 \\ -Q_0 U_0 + P_0 V_0 \end{bmatrix}$$

$$u\Omega_0 = \begin{bmatrix} L_0 \\ M_0 \\ N_0 \end{bmatrix} = S(\Omega_0) I_M \Omega_0$$

$$= \begin{bmatrix} I_{yz}(Q_0^2 - R_0^2) + Q_0 R_0 (I_{zz} - I_{yy}) + P_0 (Q_0 I_{xz} - R_0 I_{xy}) \\ I_{xz}(R_0^2 - P_0^2) + P_0 R_0 (I_{xx} - I_{zz}) + Q_0 (R_0 I_{xy} - P_0 I_{yz}) \\ I_{xy}(P_0^2 - Q_0^2) + P_0 Q_0 (I_{yy} - I_{xx}) + R_0 (P_0 I_{yz} - Q_0 I_{xz}) \end{bmatrix} \quad (14)$$

Substituting $u\Sigma_0$ and $u\Omega_0$ into Eqs. (13) yields the nonlinear equations of motion with respect to the equilibrium point as

$$\frac{d}{dt}\begin{bmatrix} \sigma \\ \omega \end{bmatrix} = \begin{bmatrix} -S(\Omega_0) & S(\Sigma_0) \\ 0 & I_M^{-1} S(I_M \Omega_0) - I_M^{-1} S(\Omega_0) I_M \end{bmatrix} \begin{bmatrix} \sigma \\ \omega \end{bmatrix}$$
$$- \begin{bmatrix} S(\omega) & 0 \\ 0 & I_M^{-1} S(\omega) I_M \end{bmatrix} \begin{bmatrix} \sigma \\ \omega \end{bmatrix}$$
$$+ \begin{bmatrix} \frac{1}{m} I_3 & 0 \\ 0 & I_M^{-1} \end{bmatrix} \begin{bmatrix} w_\sigma \\ w_\omega \end{bmatrix} + \begin{bmatrix} \frac{1}{m} I_3 & 0 \\ 0 & I_M^{-1} \end{bmatrix} \begin{bmatrix} u_\sigma \\ u_\omega \end{bmatrix} \quad (15)$$

where the relations $-S(\omega)\Sigma_0 = S(\Sigma_0)\omega$ and $-I_M^{-1} S(\omega) I_M \Omega_0 = I_M^{-1} S(I_M \Omega_0)\omega$. The associated flight control problem is to design the control force u_σ, and the control moment

u_ω so as to track the velocity command and the body-rate command Ω_0=[$P0_0$ Q_0 R_0] in the presence of the external disturbance $[w_\sigma \ w_\omega]^T$.

Nonlinear H∞ Velocity and Body-Rate Control

Comparing Eq. (15) with the standard nonlinear plant in Eq. (7a), we have

$$f(x)=\begin{bmatrix} -S(\Omega_0+\omega) & S(\Sigma_0) \\ 0 & I_M^{-1}S(I_M\Omega_0)-I_M^{-1}S(\Omega_0+\omega)I_M \end{bmatrix}\begin{bmatrix}\sigma\\ \omega\end{bmatrix} \quad (16a)$$

$$g_1(x)=g_2(x)=\begin{bmatrix} \frac{1}{m_s} & 0 \\ 0 & I_M^{-1} \end{bmatrix} \quad (16b)$$

where the state variable x is defined as $x=[\sigma^T \ \omega^T]^T=[u \ v \ w \ p \ q \ r]^T$, control $u=[u_\sigma^T \ u_\omega^T]^T=[f_x f_y f_z l m n]^T$, and disturbance $w=[w_\sigma^T \ w_\sigma^T]^T=[w_x \, w_y \, w_z \, w_l \, w_m \, w_n]^T$. Next, we need to specify the output signal z to be controlled as in the form of Eq. (7b). In this control mode, the ultimate control purpose is to track the velocity command Σ_0, and the body rate command Ω_0, and to make the tracking errors $\sigma=\Sigma-\Sigma_0$ and $\omega=\Omega-\Omega_0$as small as possible. To reflect these requirements, we choose z as

$$z=\begin{bmatrix} h_1(\sigma,\omega) \\ \rho u \end{bmatrix} \quad (17)$$

where

$$h_1(\sigma,\omega)=\left(\frac{\rho\sigma}{2}m_s\sigma^T\sigma+\frac{\rho\omega}{2}\omega^T I_M\omega\right)^{1/2} \quad (18)$$

is a measure of tracking performance; $\rho=[\rho_\sigma \ \rho_u \ \rho_x]$are weighting coefficients concerning the trade-off between tracking performance and control effort. By choosing weighting coefficients properly, it is possible to obtain an acceptably small h_1 without consuming a lot of control effort u. The problem of the H_∞, flight control design now can be stated as: find the control $u=[u_\sigma^T \ u_\omega^T]^T$ that the L_2-gain of the system is lower than γ, i.e.,

$$\frac{\int_0^T (z^T z)dt}{\int_0^T (w^T w)dt} = \frac{\int_0^T (h_1^2 + \rho_u^2 u^T u)dt}{\int_0^T (w_\sigma^T w_\sigma + w_\omega^T w_\omega)dt} < \gamma^2 \qquad (19)$$

In the above equation, a small value of γ means that the output signal z is attenuated significantly, which, in turn, implies that the deviations of the vehicle's velocity and body rate from the trim values are small with small expenditure of control effort u under the action of maneuvering targets w. It is attributed to the above inherent property of guaranteed disturbance attenuation level that H_∞, flight control can exhibit performance robustness against the variations of disturbances.

Substituting Eqs. (16) and Eq. (19) into Eq. (8), we obtain the flight control's HJPDI as

$$\begin{aligned}
&\frac{1}{2}\left(\frac{1}{\gamma^2} - \frac{1}{\rho_u^2}\right)\left[\frac{1}{m_s^2}\left(\frac{\partial V}{\partial \sigma}\right)^T\left(\frac{\partial V}{\partial \sigma}\right) + \left(\frac{\partial V}{\partial \omega}\right)^T I_M^{-2}\left(\frac{\partial V}{\partial \omega}\right)\right] \\
&+\left(\frac{\partial V}{\partial \sigma}\right)^T(-S(\omega)\sigma - S(\Omega_0)\sigma + S(\Sigma_0)\omega) \\
&+\left(\frac{\partial V}{\partial \omega}\right)^T(I_M^{-1}S(I_M\Omega_0)\omega - I_M^{-1}S(\omega)I_M\omega - I_M^{-1}S(\Omega_0)I_M\omega \\
&+\frac{1}{4}m_s\rho_\sigma\sigma^T\sigma + \frac{1}{4}\rho_\omega\omega^T I_M\omega < 0
\end{aligned} \qquad (20)$$

This is a nonlinear second-degree partial differential inequality in the unknown function $V(\sigma,\omega)$= V (u, v, w, p, q, r). If a qualified V can be found, the nonlinear , flight controller is then given by Eq. (9) as

$$u = \begin{bmatrix} u_\sigma \\ u_\omega \end{bmatrix} = -\frac{1}{\rho_u^2}g_2^T\frac{\partial V}{\partial x} = -\frac{1}{\rho_u^2}\begin{bmatrix} m_s^{-1}I_3 & 0 \\ 0 & I_M^{-1} \end{bmatrix}\begin{bmatrix} \partial V/\partial\sigma \\ \partial V/\partial\omega \end{bmatrix} \qquad (21)$$

Hence, the main problem of nonlinear H_∞, flight control design is to find a positive V satisfying the HJPDI in Eq. (20). We search for a possible quadratic solution for the nonlinear control problem in a similar form:

$$V(\sigma,\omega)=\frac{1}{2}\begin{bmatrix}\sigma^T & \omega^T\end{bmatrix}\begin{bmatrix}C_\sigma m_s I_3 & 0\\ 0 & C_\omega I_M\end{bmatrix}\begin{bmatrix}\sigma\\ \omega\end{bmatrix} \quad (22)$$

where C_σ, and C_ω, are scalar constants to be determined. Substituting Eq. (22) into Eq. (20) and performing the partial differentiations with respect to σ and ω, we get

$$\begin{bmatrix}\sigma^T & \omega^T\end{bmatrix}\begin{bmatrix}A_{11}(\omega) & C_\sigma m_s S(\Sigma_0)\\ 0 & A_{22}(\omega)\end{bmatrix}\begin{bmatrix}\sigma\\ \omega\end{bmatrix}<0 \quad (23)$$

where

$$A_{11}(\omega)=-C_\sigma m_s S(\Omega_0+\omega)+\frac{1}{4}m_s\rho_\sigma I_3+\frac{1}{2}\left(\frac{1}{\gamma^2}-\frac{1}{\rho_u^2}\right)C_\sigma^2 I_3 \quad (24a)$$

$$A_{22}(\omega)=-C_\omega S(\Omega_0+\omega)I_M+C_\omega S(I_M\Omega_0)+\frac{1}{2}\left(\frac{1}{\gamma^2}-\frac{1}{\rho_u^2}\right)C_\omega^2 I_3+\frac{1}{4}\rho_\omega I_M \quad (24b)$$

The remaining problem is to determine the values of C_σ, and C_ω, such that Eq. (23) is satisfied for arbitrary σ and ω. The difficulty appears in the functional dependence of A_1 and A_2 on ω, which destroys the quadratic structure in Eq. (23). Fortunately, this functional dependence on ω can be removed by employing the properties of the crossproduct matrix mentioned. Using these results in Eq. (23) yields

$$\begin{bmatrix}\sigma\\ \omega\end{bmatrix}^T\begin{bmatrix}\left(\frac{1}{2\gamma^2}-\frac{1}{2\rho_u^2}\right)C_\sigma^2 I_3+\frac{m_s\rho_\sigma}{4}I_3 & C_\sigma m_s S(\Sigma_0)\\ 0 & \left(\frac{1}{2\gamma^2}-\frac{1}{2\rho_u^2}\right)C_\omega^2 I_3-C_\omega S(\Omega_0)I_M+\frac{\rho_\omega}{4}I_m\end{bmatrix}\begin{bmatrix}\sigma\\ \omega\end{bmatrix}<0$$

According to the matrix inequality formula, the above equation is equivalent to

$$\frac{1}{2}\left(\frac{1}{\gamma^2}-\frac{1}{\rho_u^2}\right)C_\sigma^2+\frac{1}{4}m_s\rho_\sigma<0 \quad (25a)$$

$$\frac{1}{2}C_\omega S^T(\Omega_0)I_M+\frac{1}{2}C_\omega I_M S(\Omega_0)+\frac{1}{2}\left(\frac{1}{\gamma^2}-\frac{1}{\rho_u^2}\right)C_\omega^2 I_3+\frac{1}{4}\rho_\omega I_M$$
$$+\left[\frac{1}{2}\left(\frac{1}{\gamma^2}-\frac{1}{\rho_u^2}\right)C_\sigma^2+\frac{1}{4}m_s\rho_\sigma\right]^{-1}\left(\frac{1}{4}C_\sigma^2 m_s^2 S^2(\Sigma_0)\right)<0 \quad (25b)$$

These two inequalities are then solved together to find the ranges of C_σ, and C_ω. An explicit but sufficient condition which is found by taking the norm value for each term in Eq. (25b) can be expressed as

$$C_\sigma > \sqrt{\frac{m_s \rho_\sigma \rho_u^2 \gamma}{2(\gamma^2 - \rho_u^2)}} \tag{26a}$$

$$C_\omega > \frac{\gamma^2 \rho_u^2}{\gamma^2 - \rho_u^2}\left(\|\Omega_0\| \|I_M\| + \sqrt{\Omega_0^T \Omega_0 \|I_M\|^2 + \alpha(C_\sigma)} \right) \tag{26b}$$

where

$$\alpha(C_\sigma) = \frac{1}{2}\left(\frac{1}{\rho_u^2} - \frac{1}{\gamma^2}\right)\left(\rho_\omega \|I_M\| - \frac{4C_\sigma^2 m_s^2 \Sigma_0^T \Sigma_0}{2(\gamma^{-2} - \rho_u^{-2})C_\sigma^2 + m_s \rho_\sigma}\right)$$

As expected, the ranges of C_σ, and C_ω, are dependent on the trim conditions Ω_0 and Σ_0. However, it should be noted that the above two inequalities do not necessarily determine the lowest bounds of C_σ, and C_ω. The lowest bounds can be found numerically by searching for the minimum C_σ and C_ω satisfying Eqs. (25).

Up to this stage, we have shown that the $V(\sigma,\omega)$ given in Eq. (22) is truly a qualified solution of the HJPDI, and the two constants C_σ and C_ω in $V(\sigma,\omega)$ can be determined analytically as in Eqs. (26). After having obtained the solution $V(\sigma,\omega)$, we can compute the desired control force and moment by substituting Esq. (22) into Esq. (21).

$$u_\sigma = \begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = -\frac{1}{\rho_u^2 m_s}\frac{\partial V}{\partial \sigma} = -\frac{1}{\rho_u^2} C_\sigma \sigma = -\frac{1}{\rho_u^2} C_\sigma \begin{bmatrix} u \\ v \\ w \end{bmatrix} \tag{27a}$$

$$u_\omega = \begin{bmatrix} l \\ m \\ n \end{bmatrix} = -\frac{1}{\rho_u^2} I_M^{-1} \frac{\partial V}{\partial \omega} = -\frac{1}{\rho_u^2} C_\omega \omega = -\frac{1}{\rho_u^2} C_\omega \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{27b}$$

Although the procedures leading to the solution of HJPDI are rather involved, the resulting H_∞, control is surprisingly simple. It can be seen from Eqs. (27) that the control force u_σ, is proportional to the velocity tracking error $\sigma = \Sigma - \Sigma_0$; while the control moment u_ω, is proportional to the body-rate error $\omega = \Omega - \Omega_0$. This simple proportional feedback control can guarantee that the nonlinear flight control system is stable in the sense of Lyapunov and has L_2 gain lower than γ, as well.

Robust Test via Hard-Ware-in-the-Loop

The derived nonlinear H_∞ controller is simulated by Hard-Ware-in-the-Loop in CARCO Inc. It is noted that implementing the nonlinear H_∞ controller itself does not require the aerodynamic information of the Flight Motion Simulator. As we can see from Eqs. (27), the H_∞ control force u_σ and control moment u_ω depend on the feedback gains C_σ and C_ω which, in turn, depend on the mass m_s and moments of inertia I_M of FMS. In the following simulation first-order missile dynamics is assumed:

$$\frac{u_m}{u} = \frac{1}{\tau_d s + 1} \tag{28}$$

where u is the input acceleration command given by (27a) and (27b), and u_m is the actual missile acceleration response measured from the sensor outputs. The term τ_d is the time delay which may be subject to parameter uncertainty. The robustness of the 3D nonlinear H_∞ guidance design will be demonstrated in the following aspects: (1) robustness against variations in initial engagement condition, (2) performance with the different time delays, (3) the responses with time delay, (4) the control energy with the different time delays. (1) robustness against variations in initial engagement condition

In this part, the normalized initial angular momentum $\overline{h_0}$ is used as an index to reflect the impact of initial engagement conditions. The magnitude of $\overline{h_0}$ is between 0 and 1. It is found from Fig.2 that for any $\overline{h_0}$ between 0 and 1, i.e., for any engagement condition, the 3D H_∞ guidance design maintains excellent disturbance attenuation ability with all L_2-gains smaller than 1, while the performance becomes worse when changing the scale of the time lag, as can be seen from Fig.2.

(2) performance with the different time delays

Fig.3 shows the missile performance to maneuvering targets with time delay inputs. The ideal control command has a better performance than the controller with the actuator constraints.

(3) the responses with time delay

Fig.4 shows the missile responses to maneuvering targets with time delay constraints. The responses of the time-delay system are bounded.

(4) the control energy with the different time delays.

The RMS nondimensional control energies of two kinds of control inputs are shown in Fig.5. The lines with the actuator constraints show large control energy trades to achieve better performance.

Conclusions

In this paper the nonlinear $H\infty$ control theory has been applied to the control of general six degree-of-freedom missile flight motions. A new formulation of flight dynamics leads to missile flight control motions. The associated Hamilton-Jacobi partial differential inequalities are solved analytically, leading to nonlinear H_∞ flight control with simple proportional feedback structure. A remarkable property of the derived nonlinear H_∞ controller is that the desired control force and control moment to reject maneuvering targets can be computed quantitatively in advance without knowing the information of aero data.

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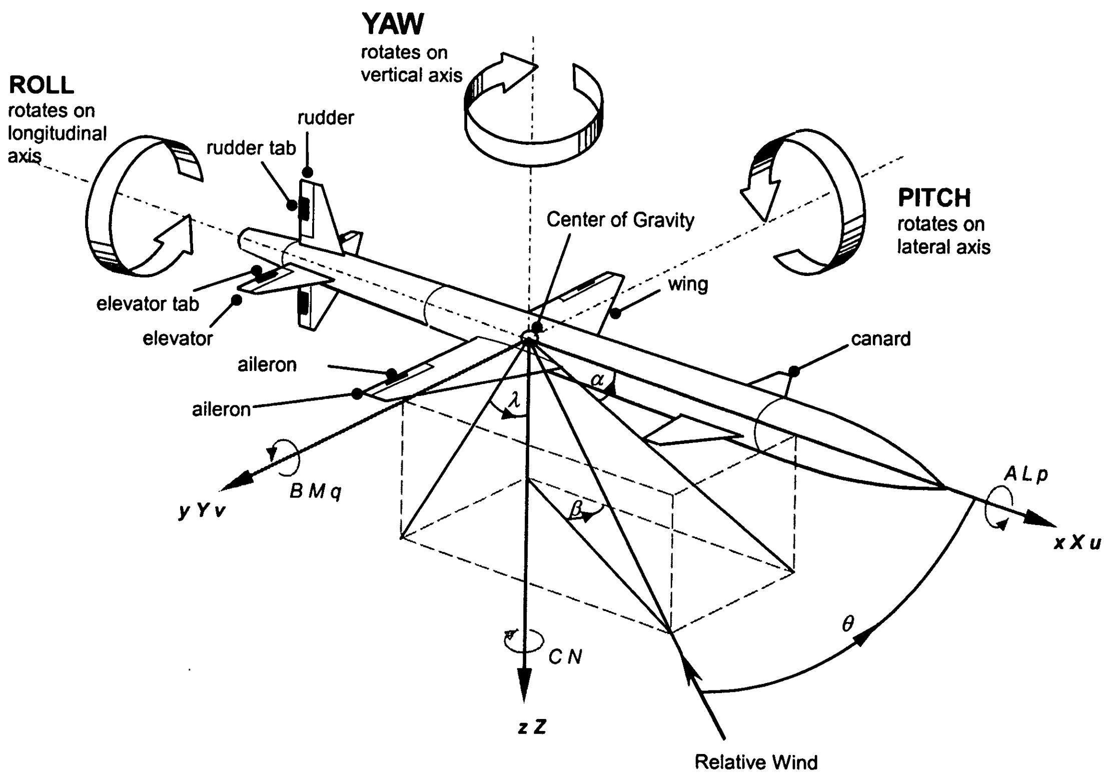

Fig.1 Definition of axes, angles, forces, and moments for missile

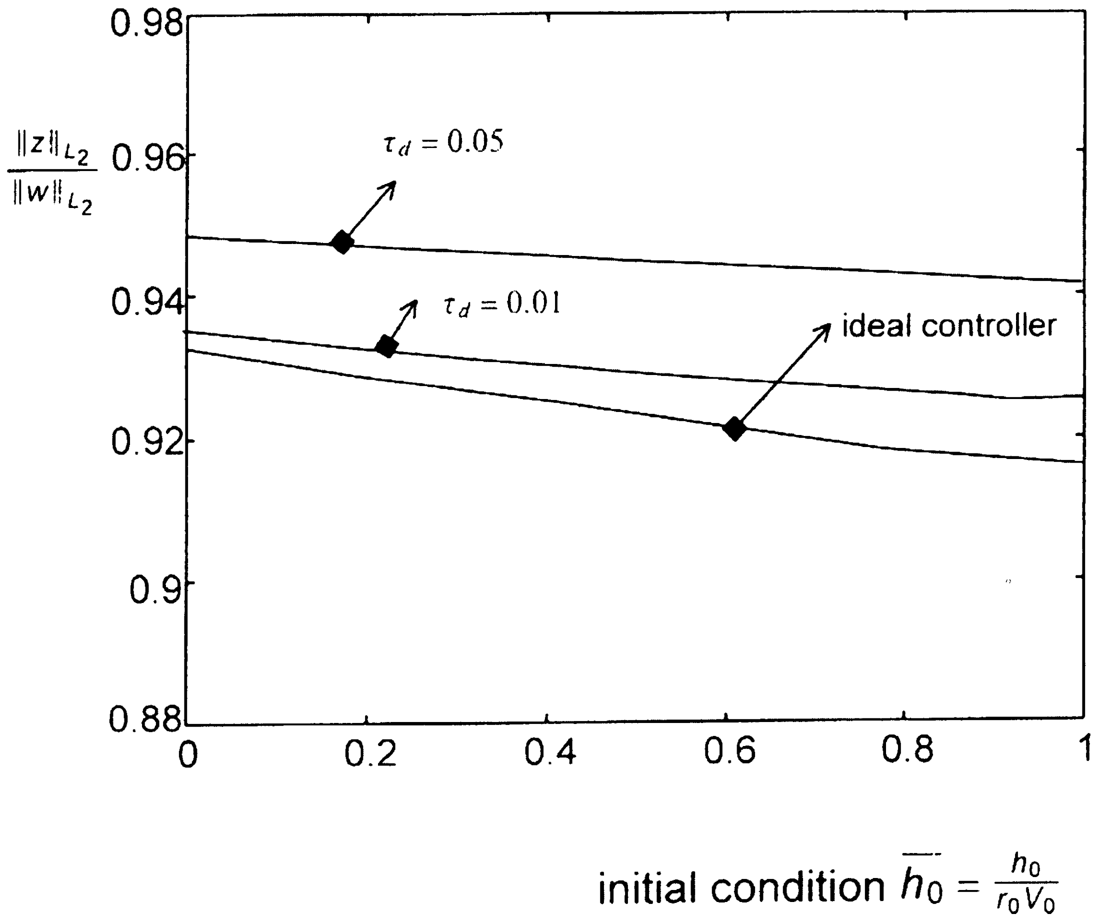

Fig.2 L_2-gain of H_∞ for guidance design for varying engagement condition

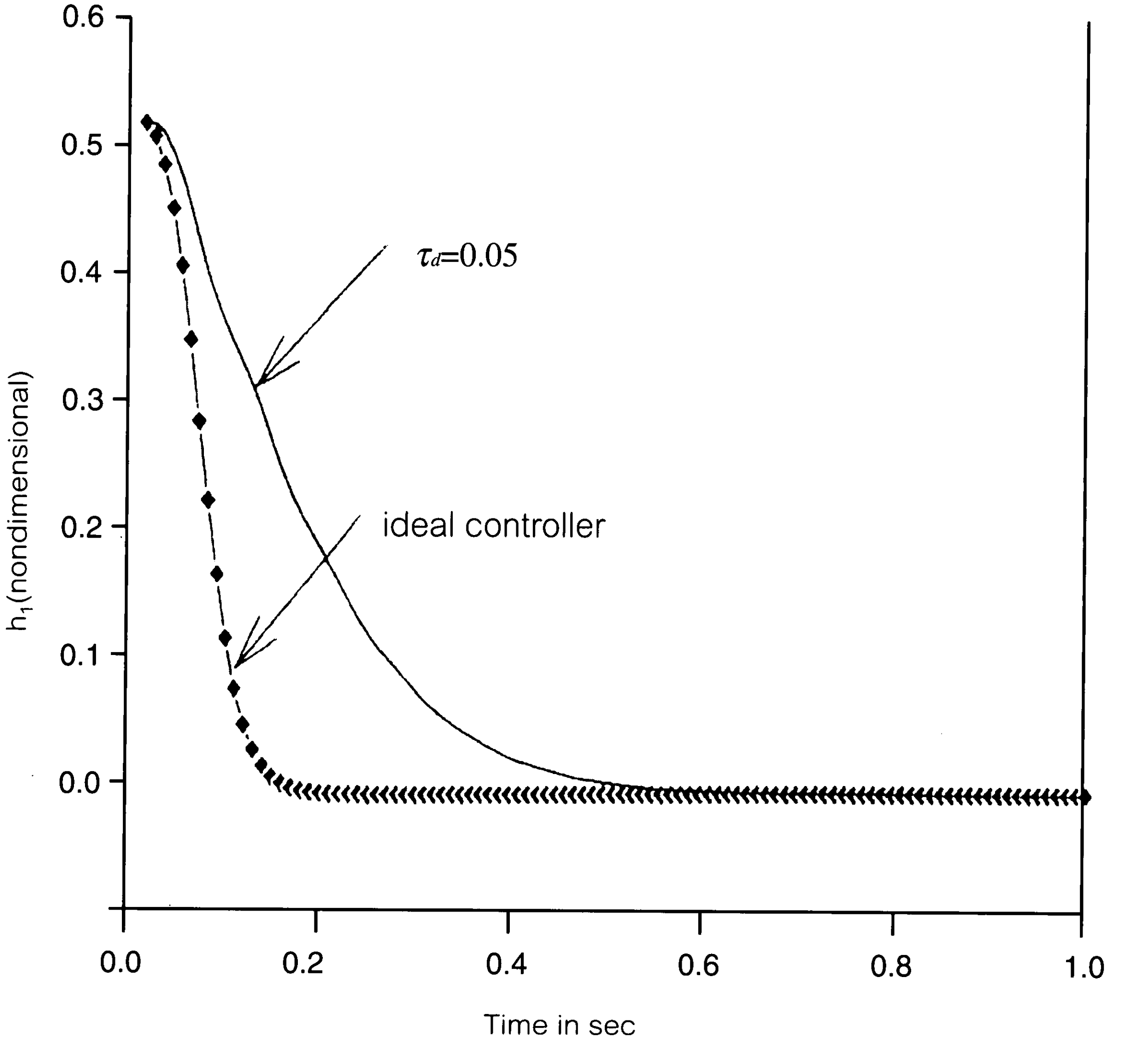

Fig.3 Performance of different control commands

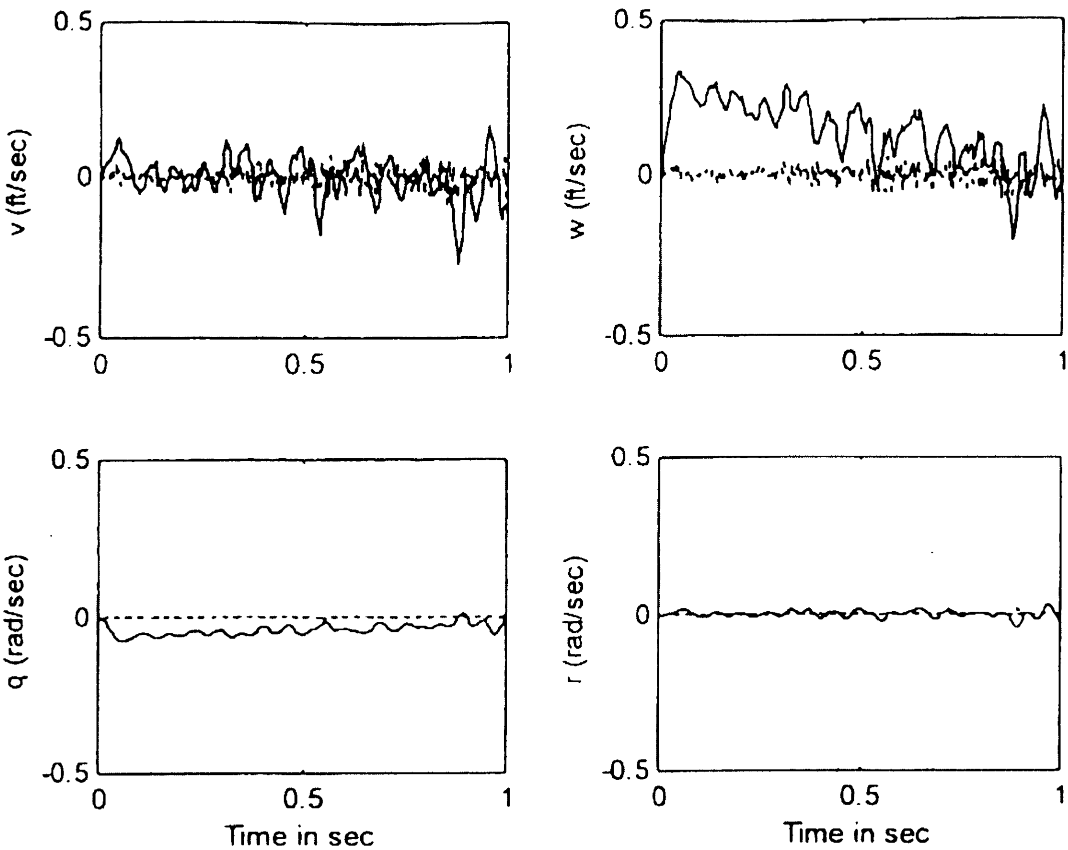

Fig.4 Responses of time delay system(τ_d=0.05)

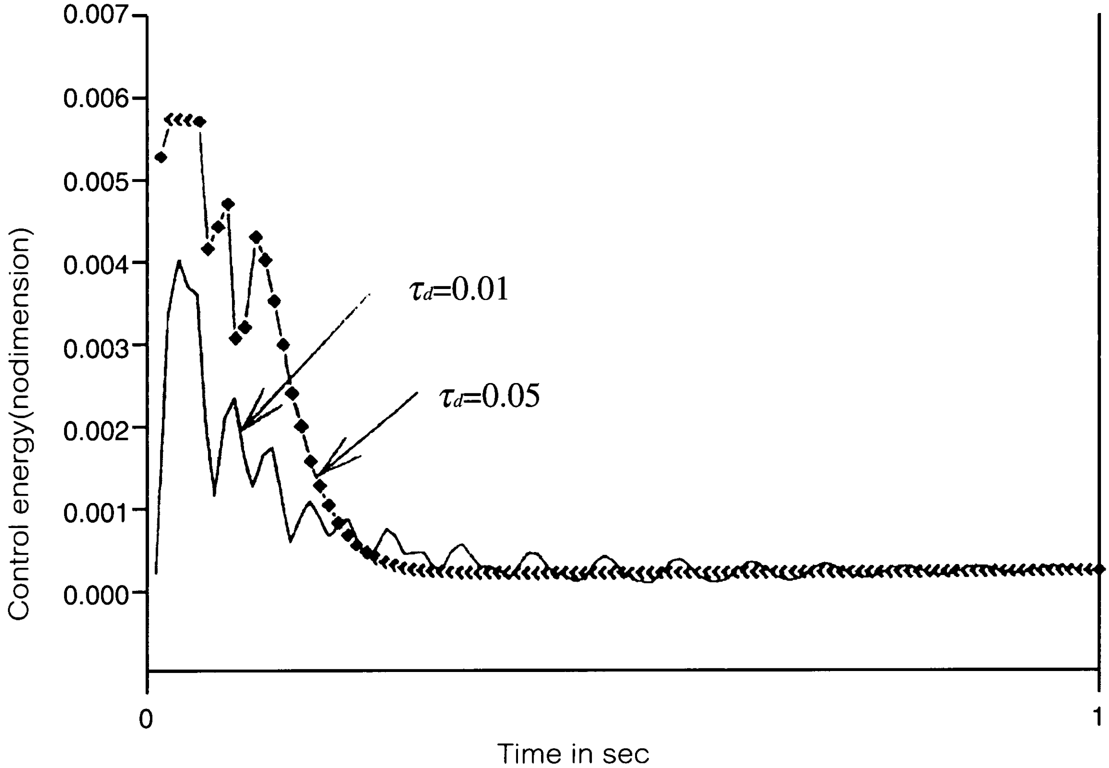

Fig.5 Control energy with the different time delay