

二維穩態不可壓縮黏性流流經具有凸塊的方形 凹孔之流場模擬

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摘要

本文主要在探討一不可壓縮黏性流流經一個具有凸塊的凹孔時，其流場的分佈影響情形。爲了清楚的了解影響流場的因素及其重要性，文中我們針對流場格點數的多寡、雷諾數的大小、凸塊的大小變化、凹孔的大小變化作一有系統的分析與比較。使我們對此類問題有更深入的了解。在分析流場中我們所採用的數值方法爲2nd upwind差分法。

關鍵字：凸塊、黏性流、差分法。

Numerical Simulation of an Incompressible Viscous Flow Over a Rectangular Cavity with Baffle

Cheng-Long Chang and Zong-Yi Lee

Abstract

This paper study incompressible viscous field flow through a rectangular cavity with baffle, its stream field of distribute consequence situation. As well as the agent and its importance of the consequence stream field. We analysis and comparison for mesh number, Reynold's number, baffle change, the dimension of the cave hole change. Analysis this article we using the numerical method is finite different method. A nonlinear differential equation problem is studied as an example. The efficiency, accuracy, and simplicity of this approach are illustrated, indicating that the proposed method can be easily extended to solve a wide range of physical engineering problems.

Keywords: baffle, Bviscous flow, Bfinite different method.

一、前言

一個具有凸塊的凹孔之流道在工程上的應用甚廣，例如氣冷式核反應器、化工程序設備及微機電子設備的散熱等。有關水平流道中具有凸塊間隔的研究有 Lehmann [1]、Agonafer[2]。Nakayama[3] 探討平板或流道上單一凸塊熱源的熱傳問題，結果顯示其特性與平滑壁的情況差異甚大。其他相關參考文獻有[4-6]。

至於凹孔中具有凸塊的研究則比較少，所以本文針對此一問題中，流場格點數的多寡、雷諾數的大小、凸塊的大小變化、凹孔的大小變化等，做一有系統的分析與比較。

二、數學模式

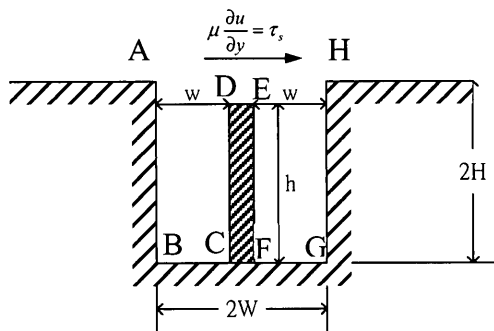


Fig. 1 物理模式

流場為二維穩態，且格點 $\Delta x = \Delta y$ 為(水平及垂直方向間隔)為可變更，由流線函數(stream-function)和渦度公式(vorticity formulation)如下：

$$\bar{u} \frac{\partial \bar{\omega}}{\partial x} + \bar{v} \frac{\partial \bar{\omega}}{\partial y} = \frac{1}{\text{Re}} \nabla^2 \bar{\omega} \quad (1)$$

$$\nabla^2 \bar{\psi} = -\bar{\omega} \quad (2)$$

其中 $\bar{\psi}$ 、 $\bar{\omega}$ 為流線函數(stream function)和渦度(vorticity)是題目所要求解的二個變數，故需解(1)(2)之聯立方程式。

$$\text{而 } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\bar{u} = \frac{\partial \bar{\psi}}{\partial y}, \quad \bar{v} = -\frac{\partial \bar{\psi}}{\partial x} \quad (3)$$

$$\bar{\omega} = \frac{\partial \bar{v}}{\partial x} - \frac{\partial \bar{u}}{\partial y} = -\frac{\partial^2 \bar{\psi}}{\partial x^2} - \frac{\partial^2 \bar{\psi}}{\partial y^2} \quad (4)$$

本文的參數設定：

$$\frac{H}{W} = 1.0, \quad \frac{h}{W} = 0.8, \quad \frac{w}{W} = 0.8$$

$$\text{Re} = \frac{\rho u_{ref} W}{\mu} = \frac{u_{ref} W}{\gamma} = 10, 100, 500, \text{ 及 } 1000。$$

令參考長度W為1

$$\frac{H}{W} = 1.0 \Rightarrow H = W = 1$$

$$\frac{h}{W} = 0.8 \Rightarrow h = 0.8W = 0.8$$

$$\frac{w}{W} = 0.8 \Rightarrow w = 0.8W = 0.8$$

I. 處理方程式

引入無因次參數：

$$x = \frac{\bar{x}}{W}, \quad y = \frac{\bar{y}}{W}, \quad u = \frac{\bar{u}}{u_{ref}}, \quad v = \frac{\bar{v}}{u_{ref}}$$

$$\gamma = \frac{\mu}{\rho}, \quad \psi = \frac{\bar{\psi}}{u_{ref}W}, \quad \omega = \frac{W\bar{\omega}}{u_{ref}}$$

由(4)式

$$\bar{\omega} = \frac{\partial \bar{v}}{\partial \bar{x}} - \frac{\partial \bar{u}}{\partial \bar{y}} = \frac{u_{ref}}{W} \frac{\partial v}{\partial x} - \frac{u_{ref}}{W} \frac{\partial u}{\partial y}$$

$$= \frac{u_{ref}}{W} \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) = \frac{u_{ref}}{W} \omega$$

將之代入(1)、(2)式作無因次化：

$$\Rightarrow (uu_{ref}) \frac{\partial \left(\frac{u_{ref}}{W} \omega \right)}{\partial (Wx)} + (vu_{ref}) \frac{\partial \left(\frac{u_{ref}}{W} \omega \right)}{\partial (Wy)}$$

$$= \frac{\mu}{\rho} \left[\frac{\partial^2 \left(\frac{u_{ref}}{W} \omega \right)}{\partial (Wx)^2} + \frac{\partial^2 \left(\frac{u_{ref}}{W} \omega \right)}{\partial (Wy)^2} \right]$$

$$\Rightarrow \left(\frac{u_{ref}}{W} \right)^2 \left(u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} \right)$$

$$= \frac{\mu u_{ref}}{\rho W^3} \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right)$$

$$\Rightarrow u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \frac{1}{Re} \nabla^2 \omega \quad (5)$$

$$\text{式中 } Re = \frac{\rho u_{ref} W}{\mu}$$

$$\Rightarrow \frac{u_{ref}}{W} \nabla^2 \psi = - \frac{u_{ref}}{W} \omega$$

$$\Rightarrow \nabla^2 \psi = -\omega \quad (6)$$

我們使用Second upwind scheme 的處理方式來離散統御方程式(5)：

$$u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \frac{1}{Re} \nabla^2 \omega$$

Second upwind scheme

$$\therefore u \left(\frac{\partial \omega}{\partial x} \right)_{i,j} = \frac{(u\omega)_{i+1/2,j} - (u\omega)_{i-1/2,j}}{\Delta x}$$

$$= \frac{(u_{i+1/2,j} + |u_{i+1/2,j}|) \omega_{i,j}}{2\Delta x}$$

$$+ \frac{(u_{i+1/2,j} - |u_{i+1/2,j}|) \omega_{i+1,j}}{2\Delta x}$$

$$- \frac{(u_{i-1/2,j} + |u_{i-1/2,j}|) \omega_{i-1,j}}{2\Delta x}$$

$$- \frac{(u_{i-1/2,j} - |u_{i-1/2,j}|) \omega_{i,j}}{2\Delta x}$$

同理

$$v \left(\frac{\partial \omega}{\partial y} \right)_{i,j} = \frac{(v\omega)_{i,j+1/2} + (v\omega)_{i,j-1/2}}{2\Delta y}$$

$$+ \frac{(v_{i,j+1/2} - |v_{i,j+1/2}|) \omega_{i,j+1}}{2\Delta y}$$

$$\frac{(v_{i,j-1/2} + |v_{i,j-1/2}|)\omega_{i,j-1}}{2\Delta y}$$

$$\frac{(v_{i,j-1/2} - |v_{i,j-1/2}|)\omega_{i,j}}{2\Delta y}$$

其中 令 $a_x = \frac{u_{i+1/2,j} + |u_{i+1/2,j}|}{2\Delta x}$

$$a_y = \frac{v_{i,j+1/2} + |v_{i,j+1/2}|}{2\Delta y}$$

$$b_x = \frac{u_{i+1/2,j} - |u_{i+1/2,j}|}{2\Delta x}$$

$$b_y = \frac{v_{i,j+1/2} - |v_{i,j+1/2}|}{2\Delta y}$$

$$c_x = \frac{u_{i-1/2,j} + |u_{i-1/2,j}|}{2\Delta x}$$

$$c_y = \frac{v_{i,j-1/2} + |v_{i,j-1/2}|}{2\Delta y}$$

$$d_x = \frac{u_{i-1/2,j} - |u_{i-1/2,j}|}{2\Delta x}$$

$$d_y = \frac{v_{i,j-1/2} - |v_{i,j-1/2}|}{2\Delta y}$$

代入方程式，整理可得

$$u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \frac{1}{\text{Re}} \nabla^2 \omega$$

$$\Rightarrow A_{i,j}\omega_{i+1,j} + B_{i,j}\omega_{i-1,j} + C_{i,j}\omega_{i,j} \\ + D_{i,j}\omega_{i,j+1} + E_{i,j}\omega_{i,j-1} = 0$$

$$\Rightarrow \omega_{i,j} = (A_{i,j}\omega_{i+1,j} + B_{i,j}\omega_{i-1,j} \\ + D_{i,j}\omega_{i,j+1} + E_{i,j}\omega_{i,j-1}) / (-C_{i,j}) \quad (7)$$

其中

$$A_{i,j} = [b_x - \frac{1}{\text{Pe}\Delta x^2}]$$

$$B_{i,j} = [-c_x - \frac{1}{\text{Pe}\Delta x^2}]$$

$$D_{i,j} = [b_y - \frac{1}{\text{Pe}\Delta y^2}]$$

$$E_{i,j} = [-c_y - \frac{1}{\text{Pe}\Delta y^2}]$$

$$C_{i,j} = [a_x - d_x + a_y - d_y \\ + \frac{2}{\text{Re}\Delta x^2} + \frac{2}{\text{Re}\Delta y^2}]$$

我們使用中央差分格式 (central difference scheme) 的處理方式來離散統

$$\nabla^2 \psi = \omega$$

御方程式(6)

$$\nabla^2 \psi = \omega$$

$$\Rightarrow \frac{\psi_{i+1,j} - 2\psi_{i,j} + \psi_{i-1,j}}{\Delta x^2} \\ + \frac{\psi_{i,j+1} - 2\psi_{i,j} + \psi_{i,j-1}}{\Delta y^2} = \omega_{i,j}$$

$$\Rightarrow \psi_{i,j} = \left(\frac{2}{\Delta x^2} + \frac{2}{\Delta y^2} \right)^{-1} \left[\frac{1}{\Delta x^2} \psi_{i+1,j} + \frac{1}{\Delta x^2} \psi_{i-1,j} + \frac{1}{\Delta y^2} \psi_{i,j+1} + \frac{1}{\Delta y^2} \psi_{i,j-1} - \omega_{i,j} \right]$$

(8)

II. 邊界處理

$$u_w = \left(\frac{\partial \psi}{\partial y}\right)_w = 0$$

$$v_w = -\left(\frac{\partial \psi}{\partial x}\right)_w = 0$$

$$\int_w -\left(\frac{\partial \psi}{\partial x}\right)_w dx = \int_w v_w dx$$

$$\Rightarrow \psi_w = -\int_w v_w dx + C$$

$$\because v_w = 0$$

$$\therefore \psi_w = \text{常數} \quad (9)$$

1. 開放邊界上：

$$\nabla^2 \psi = -\omega$$

$$\Rightarrow \omega_{\text{free surface}} = -\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right)_{\text{free surface}}$$

$$\text{因 } \psi_{\text{free surface}} = C \text{ (沿 } x\text{-方向)}$$

$$\Rightarrow \left(\frac{\partial^2 \psi}{\partial x^2}\right)_{\text{free surface}} = 0$$

$$\Rightarrow \omega_{\text{free surface}} = -\left(\frac{\partial^2 \psi}{\partial y^2}\right)_{\text{free surface}}$$

泰勒級數展開 (Taylor series expansion) :

$$\omega_{\text{free surface}} = \frac{2(\psi_{\text{free surface}} - \psi_{\text{free surface-1}})}{\Delta y^2} - \frac{2u_{\text{free surface}}}{\Delta y}$$

2. 固體邊界上：

$$\Rightarrow \omega_w = -\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right)_w$$

$$\because u=v=0$$

A. A-B 'ME-F

因 $\psi_w = C$ (沿 y -方向)

$$\Rightarrow \left(\frac{\partial^2 \psi}{\partial y^2}\right)_w = 0$$

$$\Rightarrow \omega_w = -\left(\frac{\partial^2 \psi}{\partial x^2}\right)_w$$

泰勒級數展開 (Taylor series expansion) :

$$\omega_w = \frac{2(\psi_w - \psi_{w+1})}{\Delta x^2}$$

B. C-D 'MG-H

因 $\psi_w = C$ (沿 y -方向)

$$\Rightarrow \left(\frac{\partial^2 \psi}{\partial y^2}\right)_w = 0$$

$$\Rightarrow \omega_w = -\left(\frac{\partial^2 \psi}{\partial x^2}\right)_w$$

泰勒級數展開 (Taylor series expansion) :

$$\omega_w = \frac{2(\psi_w - \psi_{w-1})}{\Delta x^2}$$

C. B-C、D-E 'MF-G

因 $\psi_w = C$ (沿 x -方向)

$$\Rightarrow \left(\frac{\partial^2 \psi}{\partial x^2}\right)_w = 0$$

$$\Rightarrow \omega_w = -\left(\frac{\partial^2 \psi}{\partial y^2}\right)_w$$

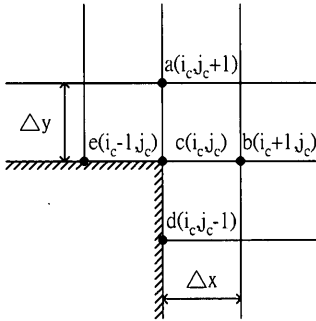
泰勒級數展開 (Taylor series expansion) :

$$\omega_w = \frac{2(\psi_w - \psi_{w+1})}{\Delta y^2}$$

3. 凸轉角尖點上 :

A. 點D

$$\omega_D = - \left[\frac{2(\psi_{i_D-1,j_D} - \psi_{i_D,j_D})}{\Delta x^2} + \frac{2(\psi_{i_D,j_D+1} - \psi_{i_D,j_D})}{\Delta y^2} \right]$$



B. 點E

$$\omega_E = - \left[\frac{2(\psi_{i_E+1,j_E} - \psi_{i_E,j_E})}{\Delta x^2} + \frac{2(\psi_{i_E,j_E+1} - \psi_{i_E,j_E})}{\Delta y^2} \right]$$

III. 收斂準則

使用相對值誤差法，並將

$$\sum_{j=1}^{J_{MAX}} \sum_{i=1}^{I_{MAX}} \frac{|f_{i,j}^{n+1} - f_{i,j}^n|}{|f_{i,j}^{n+1}|} \leq \epsilon = 10^{-5}$$

修正為

$$\sum_{j=1}^{J_{MAX}} \sum_{i=1}^{I_{MAX}} \frac{|f_{i,j}^{n+1} - f_{i,j}^n|}{|f_{i,j}^{n+1}| + \Delta\epsilon} \leq \epsilon = 10^{-5}$$

其中， $\Delta\epsilon = 10^{-5}$ 可避免產生分母為零的狀況；C

三、結果與討論

首先我們先就疊代次數作一比較討論，附表一如下：

表一：疊代次數比較圖 單位：次

格點數	21*21	41*41	61*61	81*81
Re				
10	247	834	1834	2670
100	346	1189	2255	4001
500	366	1141	2245	3824
1000	380	1041	2314	4418

格點數	101*101	121*121	141*141
Re			
10	4087	5766	7783
100	6214	8889	9630
500	5915	7796	10762
1000	7270	8419	11468

當格點數取到大約141*141以上，其實流場已經趨於穩定，表示流場圖形將不會再有很大的變化，故此表格點數只取至141*141為止。根據其間的比較可看

出當Re值相同時，格點數愈大，疊代次數愈多，疊代時間就愈久，因為疊代時間會因為電腦的速度而不同，故在此時間將不列出。其次是當格點數相同時，Re值愈大，疊代次數也愈多。

● 物理參數(不同的Re值)

緊接著對流線函數作一討論：

假設格點數121*121時是穩定的流場圖，由 Fig2~5 可以看出Re值愈大且表示上方可移動邊界的隔板抽的速度愈大(假設不考慮黏滯係數的影響)，當隔板往右邊抽的時候，一開始會先帶動上方的流場旋轉，其流動方向為順時針(速度箭頭方向)而且是一個速度很快的區域，因為其流線函數非常密集，根據流線函數的觀念其解釋如下： $\Delta\phi = VA = \text{常數}$ ，其中 ϕ 為流線函數(Fig2~5 中的每一條線)， $\Delta\phi$ 代表流量，故流線函數愈密集處流體速度愈快。這些上方擾動性強的流體會繼續帶動下方檔板兩側的流體產生逆時針方向的回流。值得注意的是當Re值大於500以上時，其左下角因為上方流體流動太強，會繼續產生另一個反方向的回流，當Re=1000時此

回流效應更強。

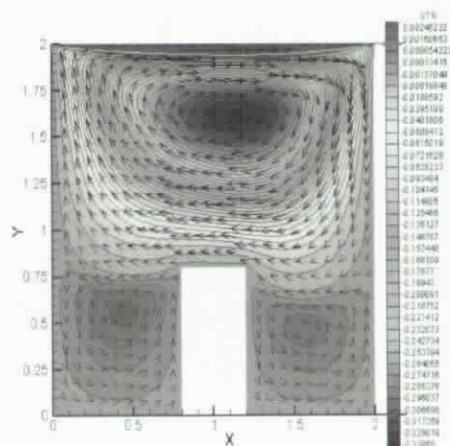


Fig2. Re=10格點數121*121

流線函數圖

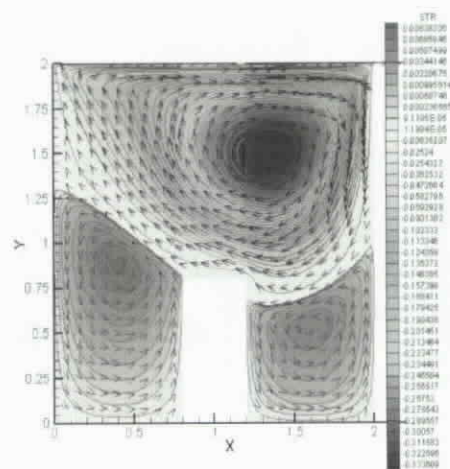


Fig3. Re=100格點數121*121

流線函數圖

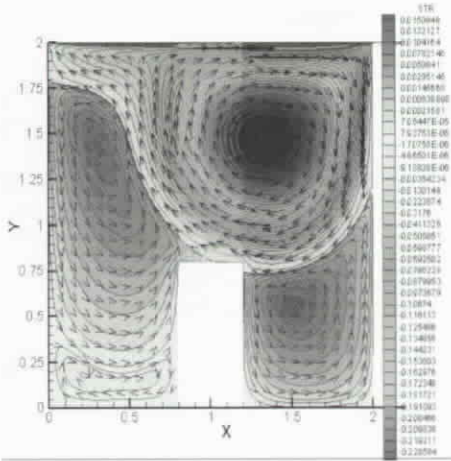


Fig4. Re=500格點數121*121

流線函數圖

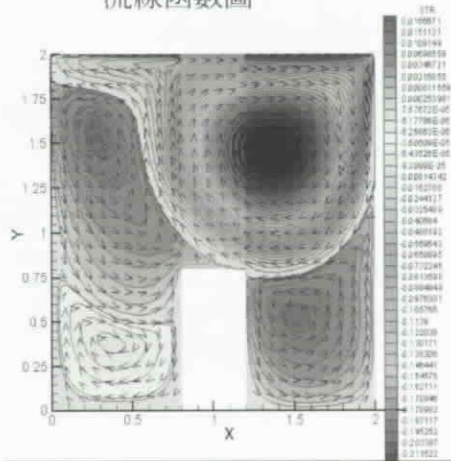


Fig5. Re=1000格點數121*121

流線函數圖

我們對不同Re下的渦度作一討論：

由Fig6~9可看出圖形中有大概四種顏色的區域，紅色跟黃色區域的值為正值代表是逆時針旋轉，以紅色值大代表旋轉效應較強，而綠色跟藍色區域的值為負值，代表是順時針旋轉，以藍色絕對值大，代表旋轉效應較強。因為渦度牽扯到旋轉的機制，假想在紅色及藍色

區域的水分子在旋轉而且旋轉的很厲害，通常在碰到檔板的時候，個別單一的水分子會因為撞擊而轉的更厲害，我們在流線函數的說明中，曾經點出上方以及下方的流場會呈繞圈的方式流動，但不一定代表水分子均在轉動，由渦度圖其實可以看出左下方的區域旋轉效應不強。

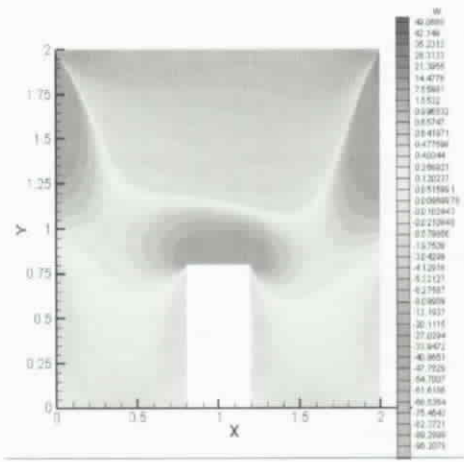


Fig6. Re=10格點數121*121渦度圖

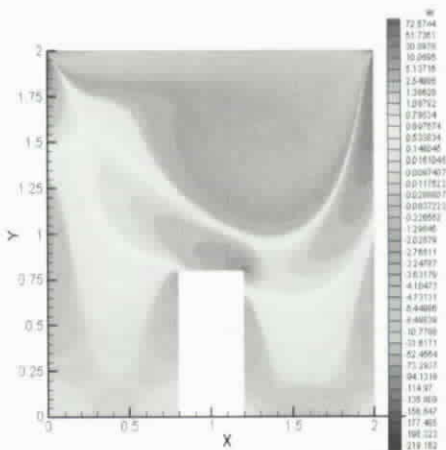


Fig7. Re=100 格點數121*121 渦度圖

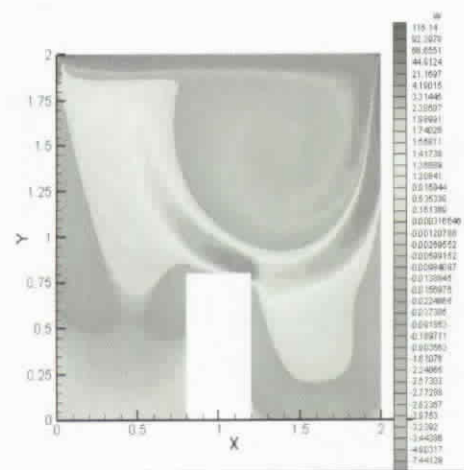


Fig8. Re=500格點數121*121 渦度圖

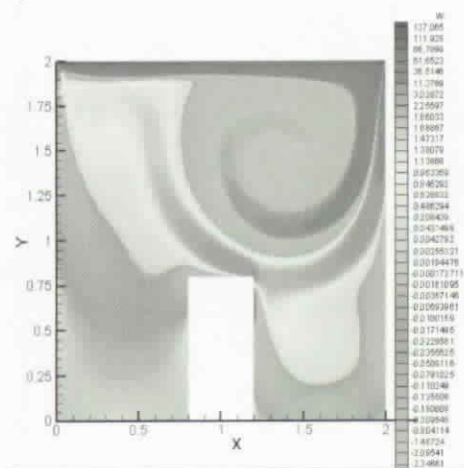


Fig9. Re=1000 格點數121*121 渦度圖

● 幾何參數(改變長、寬度)

Fig. 10 當凹孔長度變長時，會延長上方區域之流體流動現象，形成兩個大塊的區域，其流動方向相反。Fig. 11 當凹孔長度變長時，其渦度的變化情形。Fig. 12和Fig.13為凹孔長度變短

時，其流線圖與渦度圖因為長度變短會壓縮其效果。Fig.14為檔板變大時，其流線函數圖。Fig. 15檔板變大時的渦度圖我們可以看到下面區域會被壓縮，檔板處渦度大，水分子旋轉激烈。Fig. 16和Fig. 17為檔板變小時的流線圖與渦度圖，我們可以由圖得知檔板變小時較無多大變化。

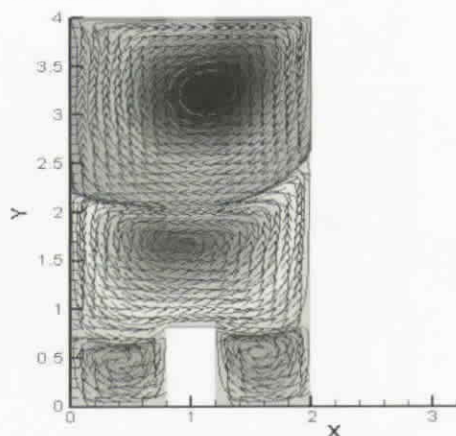
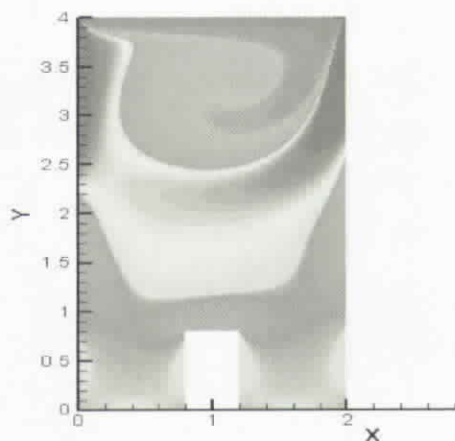
Fig.10 Re=100 凹孔長度變長
流線函數圖

Fig.11 Re=100 凹孔長度變長渦度圖

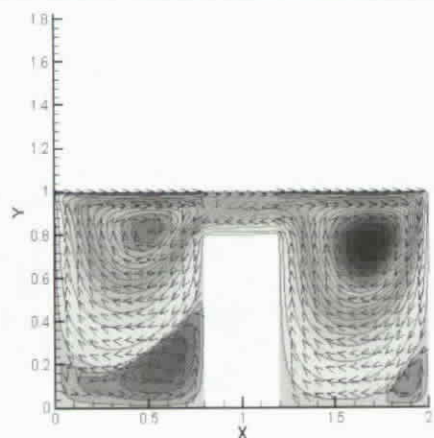


Fig.12 Re=100 凹孔長度變短
流線函數圖

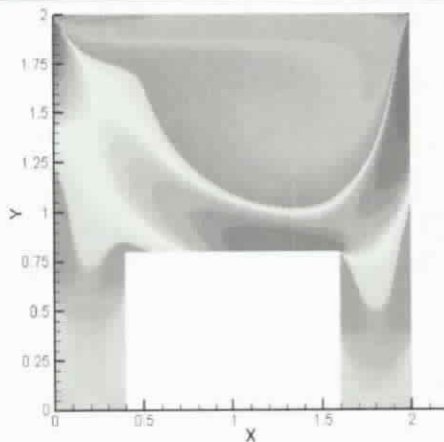


Fig.15 Re=100 檔板變大渦度圖

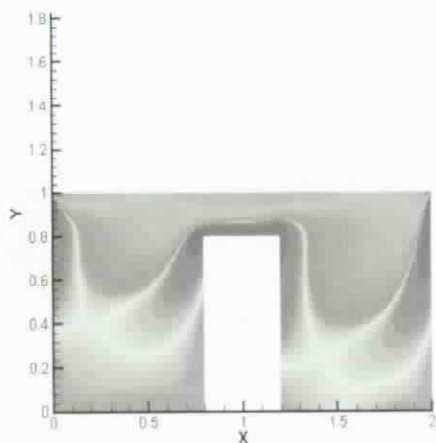


Fig.13 Re=100 凹孔長度變短渦度圖

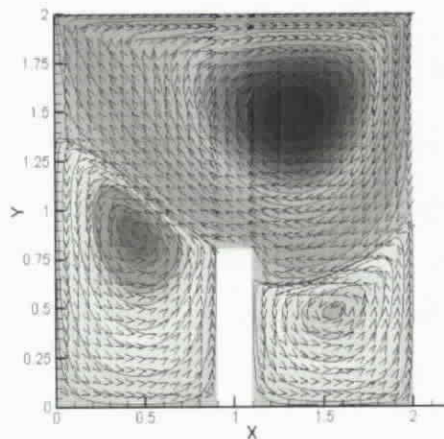


Fig.16 Re=100 檔板變小流線函數圖

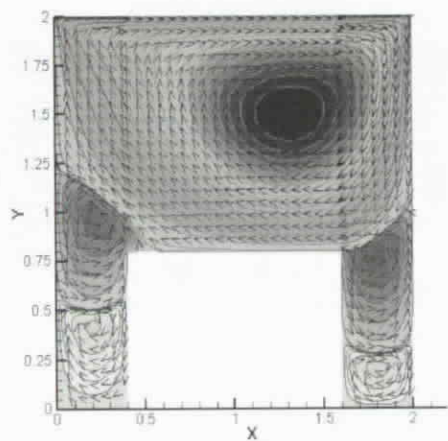


Fig.14 Re=100 檔板變大流線函數圖

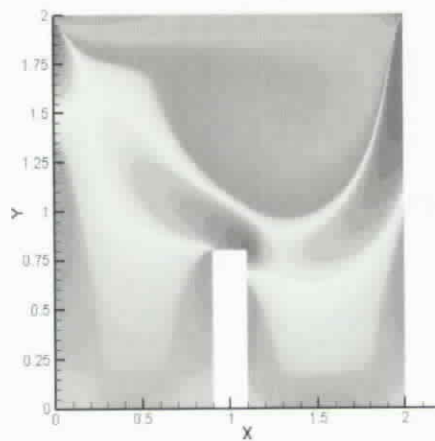


Fig.17 Re=100 檔板變小渦度圖

四、結論

本文在探討一不可壓縮黏性流，流經一個具有凸塊的凹孔後，其對影響流場的因素有了深入的了解。我們針對流場格點數的多寡、雷諾數的大小、凸塊的大小變化、凹孔的大小變化，作一有系統的分析與比較。此等結果可供日後工程應用的參考。

五、參考文獻

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