

# 火焰拉伸和路易數效應對稀薄噴霧火焰 熄滅的影響

蔡志信、侯順雄

## 摘要

一維流場中橫截面積(cross-sectional area)的改變會引發火焰拉伸(flame stretch)的作用，對預混火焰的燃燒特性具有重大的影響。但截至目前為止，相關的文獻探討僅侷限於單相氣態火焰，欠缺對於橫截面積逐漸變大或變小的一維兩相噴霧火焰分析。本研究利用高活化能極限近似微擾理論來探討具橫截面積改變的一維流場中一層流、穩態、稀薄、均一分佈噴霧火焰的燃燒特性和熄滅現象，期望了解因橫截面積變化所引起之火焰拉伸(flame stretch)、噴霧液滴半徑大小、液態燃料量和Le數(Lewis number)這四項參數對噴霧火焰的重大影響。

**關鍵詞：**稀薄噴霧，火焰拉伸，路易數，熄滅。

# Effects of Flame Stretch and Lewis Number on the Extinction of Dilute Sprays

Chih-Hsin Tsai, Shuhn-Shyurng Hou

## Abstract

The influences of flame stretch, preferential diffusion and internal heat transfer on the extinction of dilute spray flames propagating in a duct with varying cross-sectional area are analyzed using activation energy asymptotics. A completely prevaporized mode and a partially prevaporized mode of flame propagation are identified. We consider a non-conserved system in which the initial gas-phase composition is maintained the same, but the liquid fuel loading is systematically varied. Therefore, the influences of liquid fuel can be independently explored. The internal heat transfer resulted from droplet gasification, is a function of the liquid fuel loading and the initial droplet size. The analysis is restricted to a dilute spray, i.e., the amount of liquid fuel loading in the fresh mixture is so small that expansion in perturbation analysis can be performed. The results show that the internal heat transfer, associated with the liquid fuel loading of the spray, provides internal heat loss for rich sprays but heat gain for lean sprays. The burning intensities of a lean<sup>o</sup> or rich<sup>o</sup> spray is enhanced<sup>o</sup> or reduced<sup>o</sup> with increasing liquid fuel loading and decreasing initial droplet size. The positive stretch coupled with Lewis number ( $Le$ ) weakens the lean methanol-spray flame ( $Le > 1$ ) but intensifies the rich methanol-spray flame ( $Le < 1$ ). For the  $Le < 1$  flame with positive stretch or the  $Le > 1$  flame with negative stretch, no extinction occurs. A positively-stretched  $Le > 1$  flame or a negatively-stretched  $Le < 1$  flame can be extinguished by increasing the stretch. The flame stretch is found to strongly dominate the tendency for flame extinction characterized by a C-shaped curve.

**Keywords:** Dilute spray; Stretch; Lewis number; Heat loss; Extinction

## 1. INTRODUCTION

A homogeneous laminar premixed flame influenced by external heat loss can be described by a C-shaped extinction curve (a double-valued function) in the classical flame-quenching theory [1-3]. It is well known that a given combustible premixture will have two possible flame speeds under a fixed amount of heat loss: the upper branch representing stable solution; and the lower branch showing unstable solution. The extinction limit, identified by the critical point in connecting the upper and lower branch, indicates that a sufficiently large external heat loss leads to flame extinction.

Since flow stretch was further recognized as an important parameter on flame extinction [4-7], various theories [4] and experiments [5-7] on extinction characteristics were then demonstrated in the stagnation-point flow in which flow stretch is positive. It was concluded that with increasing positive stretch, burning intensity of a premixed flame is weakened

or enhanced, when the Lewis number ( $Le$ ) of mixture is larger or smaller than one, respectively. There is also a study for the propagation of a premixed flame in a close tube with varying cross-section area [8]. It was concluded that positive flame stretch increases the mass burning rate, negative flame stretch has the opposite effect, with a Lewis number larger (smaller) than one.

Studies on flame extinction introduced above were only focused on homogeneous mixture. However, the participation of fuel spray effects [9] further produced so-called internal heat loss (or gain) to the system, and thereby resulted in an S-shaped extinction curve (a triple-valued function) on spray flame extinction. Because the fuel spray absorbs heat for the gasification process, the internal heat transfer embedded in the rich and lean spray respectively resulted in heat loss and heat gain for the system. It was generally concluded that the S-shaped extinction curve is found for a rich spray, if the spray is thick enough and consists of liquid

droplets large enough. On the contrary, the flame propagation flux of a lean spray is increased with increasing liquid fuel loading and decreasing initial droplet size without the occurrence of flame extinction.

In the present study, we have formulated an extinction theory on stretched spray flames with non-unity Lewis number in a nonconserved system in which the initial gas-phase composition is maintained the same, but the liquid fuel loading is systematically varied. Therefore, the influence of liquid fuel will be independently explored without the participation of the leaning effect from the gas-phase mixture. Furthermore, the coupling effects of stretch and internal heat transfer on extinction with non-unity Lewis number will be discussed. The mathematical technique used is the matched asymptotic analysis in the limit of large activation energy. We shall also restrict our analysis to dilute sprays [10-13] in which the amount of liquid fuel loading in the total fresh mixture is very small and

can be expanded in perturbation analysis.

## Theoretical Model

We adopt a one-dimensional coordinate system in which a planar flame sits at  $x=0$  in a duct with varying cross-sectional area; the two phase combustible mixture composed of various concentrations of oxidizer, nitrogen, fuel vapor, and fuel droplets of a certain radius comes from  $x=-\infty$ ; and equilibrium reaction products move away toward  $x=+\infty$ , as illustrated in Fig. 1., a completely prevaporized mode ( $r'_i \leq r'_c$ ) and a partially prevaporized mode ( $r'_i > r'_c$ ) of flame propagation, shown in Fig. 1(a) and Fig. 1(b) respectively, are identified by a critical initial droplet size ( $r'_c$ ) for the droplet to achieve complete vaporization at the premixed flame front. We assume that the droplet will start to evaporate at  $x=x_v$ , only when the gas temperature has reached the boiling point of the liquid. Droplets then ignite upon crossing the flame, and vanish at  $x=x_e$  upon complete combustion for lean

sprays or complete evaporation for rich sprays.

We further assume that the external heat transfer being  $O(\varepsilon)$  is proportional to  $(T - T_w)$  in the upstream region of  $x_v$  to 0.  $T_w$  denotes the wall temperature in the upstream region. Since the spray is dilute and the external heat transfer is small compared with the heat release of combustion, it is reasonable to assume that the amount of liquid fuel loading and the amount of external heat transfer is of  $O(\varepsilon)$  in the asymptotic analysis. Here  $\varepsilon = T_\infty / T_a$  is the small parameter of expansion for large activation energy reactions of interest to combustion. Finally, we assume that the fuel and oxidizer reaction for the bulk premixed flame is one-step overall, that the fuel droplets burn in the flame sheet limit, and the conventional constant property simplifications apply. More detailed assumptions and comments were described in an earlier study [13].

We designate the extent of gas-phase heterogeneity by the parameter  $Z = \rho'_G / \rho'$

such that  $Z=1$  represents the completely vaporized state. Following the previous formulation [13], the present case for a duct with varying cross-sectional area can be modeled by adding  $-\rho Z u$ ,  $(1/Le)(dY/dx)$ ,  $dT/dx$  times  $(1/A)(dA/dx)$  [3] to the right-hand sides of the non-dimensional equations for gas-phase continuity, conservation of fuel, oxidizer, and energy. Where  $A$  is the cross-sectional area of the duct, and these equations are respectively given by

$$\left(\frac{d}{dx}\right)(\rho Z u) - (A_w/ZT)(1-Z_w)^{2\beta}(1-Z)^{\beta}F(T, Y_o) - (\rho Z u)(1/A)(dA/dx) \quad (1)$$

$$\left(\frac{d}{dx}\right)[\rho Z u Y_f - (1/Le)(dY_f/dx)] = W + f_f \left(\frac{d}{dx}\right)(\rho Z u) + (1/Le)(dY_f/dx)(1/A)(dA/dx) \quad (2)$$

$$\left(\frac{d}{dx}\right)[\rho Z u Y_o - (1/Le)(dY_o/dx)] = W + f_o \left(\frac{d}{dx}\right)(\rho Z u) + (1/Le)(dY_o/dx)(1/A)(dA/dx) \quad (3)$$

$$\left(\frac{d}{dx}\right)[\rho Z u T - (dT/dx)] = -W - \epsilon K(T - T_w)H(x) + f_f \left(\frac{d}{dx}\right)(\rho Z u) + (dT/dx)(1/A)(dA/dx) \quad (4)$$

where

$$A_1 = 3 \cdot (\lambda' / \rho'_G \dot{m}'_p)^2 (P' \bar{M}' / \bar{R} C'_{PG} Q_c \rho'_L) \quad (5)$$

$$W = -(B' \sigma' / \bar{M}'_0) (P' \bar{M}' / \bar{R}) \times (\lambda' / C'_{PG} \dot{m}'_p)^2 Y_o Y_f \exp(-T_a/T) \quad (6)$$

and the function  $H(x)$  in Eq.(4) is equal to 1 as  $x_v \leq x \leq 0$  or 0 as  $x > 0$  or  $x < x_v$ , while  $x = x'/l'_T$  is the non-dimensional distance expressed in units of the preheat zone thickness,  $l'_T = \lambda / (c'_{PG} \dot{m}'_p)$ . During the derivation,  $(1/A)(dA/dx)|_{x=0}$  has been stretched as  $\varepsilon \Gamma$  [8]. Here  $\Gamma$  is called the stretch parameter. In Eqs.(1)-(4), the function  $F(T, Y_o)$  and the constant parameters  $f_e, f_o,$  and  $f_T$  are respectively,  $\ln[1 + (T - T_b)/h_{LG}]$ , 1, 0 and  $-h_{LG}$  for the vaporizing droplet and  $\ln[1 + (T - T_b - Y_o)/h_{LG}]$ , 0, 1, and  $(1 - h_{LG})$  for the burning droplet.  $K$  represent the heat transfer coefficient for the external heat transfer in the upstream region. In this study, we assume  $T_{-\infty} = T_u$  for simplification.

Performing the inner and outer expansions based on the small parameter of  $\varepsilon$ , and following the detailed matching procedure of the previous study [13] to match the inner and outer solutions, we therefore reach the final results as follows:

$$\dot{m}_o'^2 = \exp[T_1^+(0)] \quad (7)$$

Equation (7) indicates that the flame propagation flux is exponentially affected by the first-order temperature downstream near the flame. The first-order temperature  $T_1^+(0)$  is expressed by the following equation:

$$T_1^+(0) = T_{1\infty} + \left(\frac{\dot{m}_o' \gamma}{T_{\infty}}\right) (T_{\infty} + L - \alpha) \int_0^{x_v} \xi_0^+ e^{-\dot{m}_o' u} du \quad (8)$$

For the sake of notation compactness, we use  $\alpha = 1$  for lean sprays and  $\alpha = 0$  for rich sprays. The liquid fuel loading is represented by  $\gamma$  through the expansion of  $Z_{-\infty} = 1 - \varepsilon \gamma$  for dilute sprays [13].

For completely prevaporized sprays, the value of  $x_e$  is equal to zero, we obtain

$$\dot{m}_o'^2 = \exp[T_{1\infty}] \quad (10)$$

where

$$T_{1\infty} = \frac{\gamma}{T_{\infty}} [\alpha - L + (T_b - T_{\infty})] - \frac{c'_L}{c'_{PG}} (T_b - T_{\infty}) - \frac{\Gamma Y_{i\infty}}{\dot{m}_o' T_{\infty}} \left(1 - \frac{1}{Le}\right) - \frac{KY_{i\infty}}{\dot{m}_o'^2 T_{\infty}} \left(1 - \frac{T_b - T_{\infty}}{T_{\infty} - T_{\infty}}\right) \quad (10)$$

Here we use  $i = F$  for lean sprays and  $i = O$  for rich sprays. Equation (9) also

indicate that the flame flux is independent of the initial droplet size of the spray.

On the basis of the formulated results, Eqs. (7) and (9), sample calculations for methanol burning in air are now considered in a nonconserved manner which maintains the initial gas-phase composition not varies the liquid fuel loading. We adopt an external heat loss parameter  $Q_L = K(T_\infty - T_b)/T_\infty$  which is similar to that of the earlier studies [10-11]. The influence of flow stretch and preferential diffusion on dilute spray flames in the problem will be assessed based on four parameters, namely the initial droplet radius ( $r'_0$ ), the liquid fuel loading ( $\gamma$ ), flow stretch ( $\Gamma$ ), and Lewis number (Le). Here  $r'_0$  and  $\gamma$  show the internal heat transfer (heat gain or heat loss) for the fuel spray. Lewis number is defined as  $\lambda' / (\rho'_0 c'_{p0} D'_j)$  in which the diffusion coefficient of the deficient reactant in the mixture is used and variables are determined based on the mean gaseous temperature upstream of the flame. Methanol-air premixture of  $\Phi_G = 0.8$  and

$\Phi_G = 1.5$ , corresponding to  $Le = 1.0371$  and  $0.9477$ , respectively, are adopted to show the influence of nonunity Lewis number.

### Lean spray flame with $Le > 1$

We first investigate the completely prevaporized sprays ( $r'_i \leq r'_c$ ) in which no liquid droplets exist downstream of the flame. Fig. 2 demonstrates the flame flux ( $\dot{m}_0$ ) of lean methanol-spray flame with no heat loss ( $Q_L = 0$ ) as functions of  $\Gamma$  and  $\gamma$ . In the region of positive stretch, the increase of flow stretch results in the decrease in flame propagation flux for  $Le > 1$ . However, when the flame experiences negative stretch, the flame extinction would never occur. Therefore, the upper and lower branches of the C-shaped extinction curves correspond to the stable and unstable solutions, respectively, and are connected at critical points represented by the symbol  $\bullet$ . The critical points are identified as points of flame extinction. For a given  $\gamma$ , the increase of  $\Gamma$  first leads to a decrease of  $\dot{m}_0$  indicating that a larger

stretch lead to a more weakened flame, and finally results in flame extinction when the flow stretch is large enough. This is mainly resulted from the suppression of burning intensity by flow stretch for an  $Le > 1$  flame. In the contrary, when the stretch is negative, the decrease of  $\Gamma$  would never extinguish the flame because the negative stretch would strengthen the burning intensity.

Considering the partially prevaporized sprays ( $r'_i > r'_c$ ), the influence of the initial droplet size on flame characteristics is shown in Fig. 3 for a lean methanol-spray flame of  $\Phi_g=0.8$ ,  $\gamma=0.04$ , and  $Le=1.0371$ . Fig. 3 shows that with increasing the initial droplet size, the upper branch corresponding to the stable solution for a partially prevaporized spray first deviates from that for the completely prevaporized spray ( $r'_i \leq r'_c$ ), and approaches that for a homogeneous mixture ( $\gamma = 0$ ). This indicates that the flame flux decreases with increased initial droplet size or flow stretch. The former is due to the

reduction of internal heat gain; the latter is caused by the augmentation of the  $Le > 1$  effect. A lean spray containing larger droplets will have weaker prevaporization upstream of the flame and provides a smaller amount of internal heat gain, and therefore has a diminished burning intensity. Hence, it can be extinguished by a smaller flow stretch.

### Rich spray flames with $Le < 1$

Fig. 4 shows the flame propagation flux  $\dot{m}_0$  of rich methanol-spray flames of  $\Phi_g=1.5$  and  $Le = 0.9477$  as functions of  $\Gamma$  and  $\gamma$  under completely prevaporized sprays with no heat loss. Contrary to the lean spray, the liquid fuel absorbs heat for upstream prevaporization, producing the secondary gasified fuel which is equivalent to the inert substance with no contribution to burning for a rich spray, thus providing an overall internal heat loss, and weakening the flame propagation flux. For a given  $\Gamma$ , the increase of  $\gamma$  leads to decrease in  $\dot{m}_0$  because a larger  $\gamma$  absorbs a larger amount



of heat from flame for upstream droplets evaporation representing a larger heat loss.

For a given  $\gamma$ , the decrease of  $\Gamma$  leads to decrease of  $\dot{m}_o$ , and finally results in flame extinction represented by the symbol ● which is the critical point by connecting the upper and lower branches. However, when the flame with positive stretch, the flame flux increases with increased stretch for the  $Le < 1$  effect.

For partially prevaporized sprays, the effects of  $\Gamma$  on flame flux with various initial droplets size for a rich spray is shown in Fig. 5. Fig. 5 shows that with increasing the initial droplet size, the flame flux first deviates from that for the completely prevaporized spray ( $r'_i \leq r'_c$ ), and approaches that for a homogeneous mixture ( $\gamma = 0$ ). This indicates that the flame flux increases with increased initial droplet size or flow stretch. The former is due to the reduction of internal heat loss; the latter is caused by the enhancement of the  $Le < 1$  effect. A rich spray containing larger droplets endures a weaker

prevaporization upstream leads to a smaller amount of internal heat loss, and therefore has an enhanced burning intensity.

## Conclusion

Following activation energy asymptotics, an extinction theory of stretch premixed flames with combustible sprays was developed to explore the influence of liquid fuel spray, flow stretch, and Lewis number on the flammability limit and extinction of methanol sprays. Results are summarized as follows:

1. The flow stretch weakens and strengthens the burning intensity of the  $Le > 1$  flame (lean methanol flame) and the  $Le < 1$  flame (rich methanol flame), respectively.
2. For the lean methanol-spray flame with  $Le > 1$ , the burning intensity weakened by the flow stretch can be enhanced when the lean spray has a larger amount of liquid fuel loading or a smaller initial droplet size.
3. For the rich methanol-spray flame with

Le < 1, the burning intensity weakened by the flow stretch can be enhanced when the rich spray has a smaller amount of liquid fuel loading or a larger initial droplet size.

## Reference

- [1] Spalding, D.B., *Pro. Roy. Soc., A* 240 (1957), p.83
- [2] J.D. Buckmaster, *Combust. Flame* 26(1976) p.151
- [3] J.D. Buckmaster and G..S.S Ludford, *Theory of Laminar Flame*, p.38, Cambridge University Press, Cambridge, England (1982)
- [4] Kim, Y.D and Matalon, M., *Combust. Flame* 73:303~313 (1988)
- [5] Ishizuka, I. And Law, C.K., in *Nineteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1982, pp. 327~335
- [6] Sato, J., *Nineteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1982, pp.1541~1548
- [7] Tsuji, H. and Yamaoka, I., in *Nineteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, 1982, pp.1533~1540
- [8] J. H. Tien., *Combust. Flame* 107:303~306(1996)
- [9] Huang, C. L., Chiu, C. P., and Lin, T. H., *J. Chinese Soc. Mech. Eng.* 10:333~343(1989)
- [10] Liu, C.C. and Lin, T.H., *Combust. Flame*, vol.85 (1991), p.468
- [11] Hou, S.S, Liu, C.C. and Lin, T.H., *Int. J. Heat Mass Transfer*, vol.36, No.7(1993), p.1867
- [12] Hou, S.S. and Lin, T.H., *Atomization and Sprays*, vol.9 (1999), p.355
- [13] Lin, T.H., Law, C.K. and Chung, S.H., *Int. J. of Heat and Mass Transfer*, vol.34, No.5 (1988), p.1023

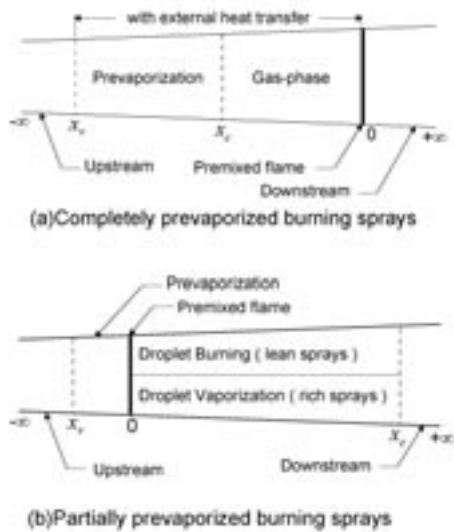


Fig.1 Schematic diagram of (a) completely prevaporized, and (b) partially prevaporized burning sprays

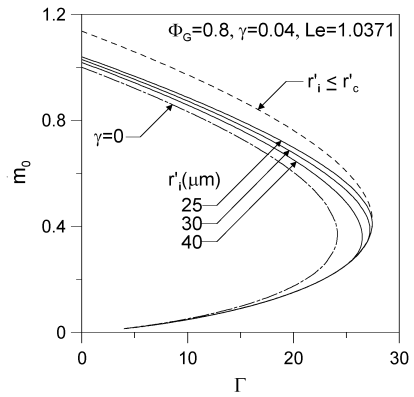


Fig. 3. Flame flux ( $\dot{m}_0$ ) as a function of the flow stretch ( $\Gamma$ ) with various values of  $r'_i$  for a lean spray flame

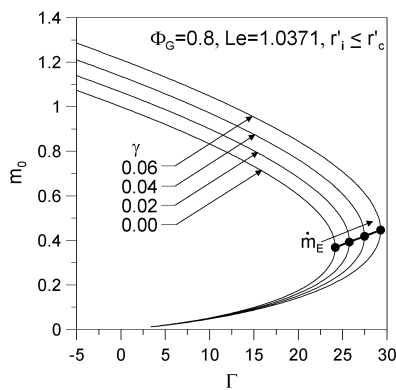


Fig. 2. variations of the flame flux ( $\dot{m}_0$ ) with the flow stretch ( $\Gamma$ ) and the liquid fuel loading ( $\gamma$ ) for a lean spray flame

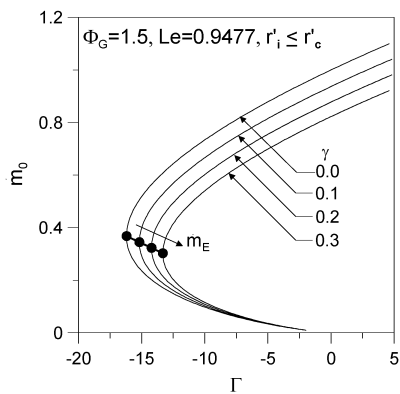


Fig. 4. variations of the flame flux ( $\dot{m}_0$ ) with the flow stretch ( $\Gamma$ ) and the liquid fuel loading ( $\gamma$ ) for the rich spray flame

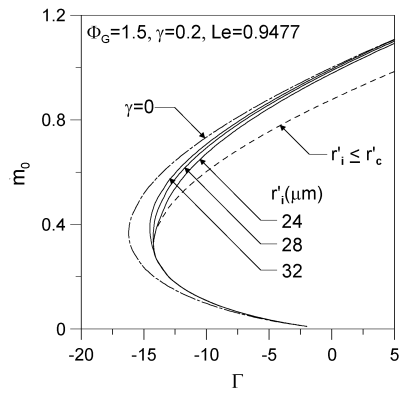


Fig. 5. Flame flux ( $\dot{m}_0$ ) as a function of the flow stretch ( $\Gamma$ ) with various values of  $r'_i$  for a rich spray flame