The Nonlinear Decoupling Control of the Mobile Platform

Pao-Hwa Yang, Shiu-Ping Wang, Che-Chia Chang, Chun-Hung Pai

Abstract

In this paper, a study of the control mobile platform by the nonlinear decoupling technique is proposed. Due to the wheeled platform is constrained to some nonholonomic conditions, a coupling matrix exists in the control input of the equation of motion, which will cause some difficulties in designing a controller based on the independent input. By providing a pseudo-inverse matrix to eliminate the coupling term and then employing the two loops design, the tracking issue of the mobile platform can be addressed.

Keywords: nonholonomic conditions, pseudo-inverse matrix, two loops design.

Pao-Hwa Yang, Assistant Professor, Department of the Mechanical Engineering, HIT.
Shiu-Ping Wang, Assistant Professor, Department of the Mechanical Engineering, HIT.
Che-Chia Chang, Graduate Student, Department of the Mechanical Engineering, HIT.
Chun-Hung Pai, Graduate Student of the Department of the Mechanical Engineering, HIT.
投稿日期：981002 接受刊登日期：981029

移動平台非線性去耦化控制設計

楊伯華、王旭平、張哲嘉、白峻宏

摘 要

本文旨在提出移動平台使用非線性去耦化控制研究，由於移動平台受限於 nonholonmic 條件，使得其運動方程式在輸入項存有一個偶合矩陣，這對於以獨立輸入做控制器設計爲基底的方法，造成困難。經提出擬反矩陣法消除偶合矩陣，再使用双回路設計，使得移動平台追蹤目標設計得以完成。

關鍵詞：nonholonmic 條件、擬反矩陣法、双回路設計。

楊伯華：修平技術學院機械系助理教授
王旭平：修平技術學院機械系助理教授
張哲嘉：修平技術學院精密機械研究所研究生
白峻宏：修平技術學院精密機械研究所研究生

I. Introduction

The discussion of the controlling a wheeled mobile platform can be traced back to a decade before [1-4]. Many researchers study this problem from modeling a wheeled mobile platform. In order to obtain the dynamic model of the mobile platform, a simple mechanical structure is employed, which a platform with two wheels in the rear to provide a driving torque and one wheel in the front to give a direction control. With the Lagrange formulation and the kinematic nonholonmic constraints such as no slipping occurs between the wheels and ground and the mobile can not move in lateral direction, the dynamic equation of the mobile platform can be constructed. Due to the dynamic behavior of the wheeled platform is confined to the nonholonomic constraints; the dynamic equations of the mobile platform will be nonlinear and parameter-dependent. However, the issue about how to solve this nonlinear equation to find a controller such that the mobile platform can follow the designated path still attracts researcher's attention. Among them, Yamamoto [2-4] focused on the dynamic behavior of a moving platform and the manipulator position. As the output equation is the final position of the manipulator, the command following problem can be rephrased as the trajectory tracking problem. Shen Lin and Goldernberg proposed a neural-network control of mobile manipulator [9-10]. Using neural-network control with on-line learning algorithms, they can design a robust-adaptive controller without requiring off-line tuning. Fierro and Lewis [7] also design a controller by application of neural-network theory. However, they combine adaptive backstepping and Lyapunov stability theory to develop so called a kinematic/torque control law to design a controller. By using this technique, they can deal with more control problems such as: tracking a reference trajectory, path following and stabilization about a desired posture. Y. Kanayama [11] and C. Samson [12-13] provided a nonlinear feedback controller design.

However, most results of the previous study to design the controller starts from the error model then derived the velocity tracking as this error terms. An alternative design is proposed in this paper, which is simpler and effect to deal with a complicated dynamic equation. Due to a wheeled platform is constrained to some nonholonomic conditions, a coupling matrix exists in the control input term of the motion equation, which will cause some difficulties in constructing a controller to those design based on the independent input. By providing a pseudo-inverse matrix to eliminate the coupling term and then employing the two loops design, a tracking issue of the mobile platform can be addressed.

This paper is organized as follows. Section II is the introduction of the dynamic model of the mobile platform. The decoupling matrix is constructed in section 3. Section 4 presents the loop control design to address the tracking issue. Simulation result is presented in section 5. Final remark is in section 6.

II. The dynamic model of a wheeled mobile platform

In this section, the motion equations and constraint equation of a wheeled mobile platform will be derived.

In figure 1, the point $c(x,y)$ is in the mass center of the platform respected to the frame $X-Y$. The nonholonomic constraint is stated as no slipper occurs between the wheels and ground which is in terms of the following equation

$$\dot{y}\cos\theta - \dot{x}\sin\theta - d\dot{\theta} = 0 \quad (1)$$

or

$$A(q)\dot{q} = 0 \quad (2)$$

where $A(q) = \begin{bmatrix} -\sin\theta & \cos\theta & -d \end{bmatrix}$ and $q = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$.

The dynamic equation is derived from the Lagrange formalism as the following:

The total energy of the mobile platform is the kinetic energy. Then the Lagrangian

$$L = K = \frac{1}{2}\dot{q}^T M(q)\dot{q} \quad (3)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = \tau_i - a_{1i}\lambda_1 - a_{2i}\lambda_2 \cdots, i = 1, \ldots n$$

τ_i: the input, a_{1i}, a_{2i} :the constraint constant, λ_1, λ_2 :the multipliers.

or in matrix representation:

$$\frac{d}{dt}\left(\frac{\partial K}{\partial \dot{q}}\right) - \frac{\partial K}{\partial q} = \tau - A^T(q)\lambda \quad (4)$$

As results, the equation of motion of a wheeled mobile platform with nonholonomic constraints can be established as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + F(q,\dot{q}) = E(q)\tau - A^T(q)\lambda \quad (5)$$

and the kinematics relation

$$\dot{q} = S(q)v(t) \quad (6)$$

In which, $q \in \Re^n$ is the generalized coordinates , $M(q)$ is $n \times n$ inertial matrix, $C(q,\dot{q})$ is $\Re^{n\times n}$ the centripetal and coriolis matrix, $F(q,\dot{q}) \in \Re^n$ is the friction and gravity matrix, $E(q)$ is $n \times r$ input matrix, τ is r dimensional input vector, $A(q)$ is $m \times n$ Jacobin matrix and λ is the vector constrain force. $v(t)$ is the velocity vector of the mobile in the mass center c, $S(q)$ is the transition matrix between the

moving frame and the inertial frame.
For more detailed results, interesting reader can refer to [3, 14].

As more specified, the structure of the mobile platform is given as the following:

$$M(q)=\begin{bmatrix} m & 0 & md\sin\theta \\ 0 & m & -md\cos\theta \\ md\sin\theta & -md\cos\theta & I \end{bmatrix}$$

$$C(q,\dot{q})=\begin{bmatrix} 0 & 0 & md\dot{\theta}\cos\theta \\ 0 & 0 & md\dot{\theta}\sin\theta \\ 0 & 0 & 0 \end{bmatrix}$$

m : the mass of the platform

d : the distance between COM and the center axis of
the wheel.

I : the moment of the inertial.

$$E(q)=\frac{1}{\rho}\begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ R & -R \end{bmatrix} \quad \tau=\begin{bmatrix} \tau_r \\ \tau_\ell \end{bmatrix}$$

ρ : the radius of the wheel.

R : the distance from the center axis to the wheel.

τ_r, τ_ℓ represent the control torque in right and left wheels respectively.

$$A(q)^T=\begin{bmatrix} -\sin\theta \\ \cos\theta \\ -d \end{bmatrix}$$

$$\lambda=-m(\dot{x}\cos\theta+\dot{y}\sin\theta)\dot{\theta}$$

The (6) is expressed as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}=\begin{bmatrix} \cos\theta & -d\sin\theta \\ \sin\theta & d\cos\theta \\ 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} v \\ w \end{bmatrix} \tag{7}$$

In which v represents the forward linear velocity and w is the angular velocity.

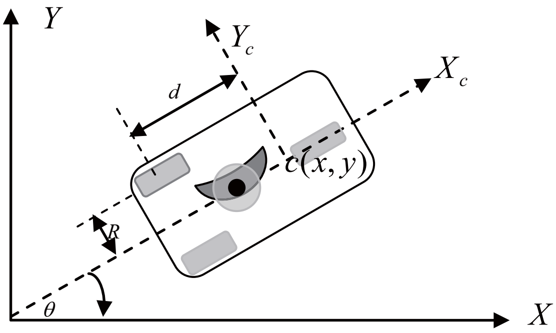

Figure 1. The coordinate of the mobile platform

In figure1, (X_c, Y_c) is the Cartesian coordinate of the center of mass (COM) of the mobile platform and θ is the orientation of the platform.

III.Construction of a seudo-inverse matrix

A theorem is furnished to construct a pseudo-inverse matrix as the following.

Theorem 1: Moore-Penrose pseudo inverse

If a matrix $A(m\times n)$ is full rank, the Moore-Penrose pseudo inverse can be directly calculated as follows:

1. $m<n: A^{+}=A^{T}(AA^{T})^{-1}$

2. $m > n : A^{+} = (A^{T} A)^{-1} A^{T}$

From (5), there is a matrix $E(q)$ coupling with the control input τ, which will cause some difficulties in designing a controller based on the independent input. By applying theorem 1, a pseudo-inverse matrix can be constructed as $\psi(q) = \frac{\rho}{2}\begin{bmatrix} \cos\theta & \sin\theta & \frac{1}{R} \\ \cos\theta & \sin\theta & -\frac{1}{R} \end{bmatrix}$ such that $\psi(q) \cdot E(q) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Therefore, multiply (5) by $\psi(q)$ and neglect the friction and gravity term $F(q,\dot{q})$, (5) can be recast as

$$\psi(q) \cdot M(q)\ddot{q} + \psi(q) \cdot C(q,\dot{q})\dot{q} = \tau - \psi(q) \cdot A^{T}(q)\lambda \tag{8}$$

, or simplified notation as

$$\overline{M}(q)\ddot{q} + \overline{C}(q)\dot{q} + \overline{A}(q)\lambda = \tau \tag{9}$$

$$\overline{M}(q) = \begin{bmatrix} \frac{m\rho\cos\theta}{2} + \frac{dm\rho\sin\theta}{2R} & \frac{m\rho\sin\theta}{2} - \frac{dm\rho\cos\theta}{2R} & \frac{I\rho}{2R} \\ \frac{m\rho\cos\theta}{2} - \frac{dm\rho\sin\theta}{2R} & \frac{m\rho\sin\theta}{2} + \frac{dm\rho\cos\theta}{2R} & -\frac{I\rho}{2R} \end{bmatrix}$$

$$\overline{C}(q) = \begin{bmatrix} 0 & 0 & \frac{dm\rho\dot{\theta}}{2} \\ 0 & 0 & \frac{dm\rho\dot{\theta}}{2} \end{bmatrix}$$

$$\overline{A}(q) = \begin{bmatrix} -\frac{d\rho}{2R} \\ \frac{d\rho}{2R} \end{bmatrix}$$

Because, $\overline{M}(q)$ is not a square matrix, if a pseudo inverse matrix $\overline{M}^{+}(q)$ can be found, then (9) is rewritten as the following result,

$$\ddot{q} = \overline{M}^{+}(q)(\tau - \overline{C}(q)\dot{q} - \overline{A}(q)\lambda) \tag{10}$$

and $\overline{M}^{+}(q)$ can be obtained again from the theorem 1.

In our case, the structure of $\overline{M}^{+}(q)$ is the following.

$$\overline{M}^{+}(q) = \begin{bmatrix} \frac{(I^2 + d^2m^2)\cos\theta + dm^2R\sin\theta}{Id^2m\rho + d^2m^3\rho} & \frac{(I^2 + d^2m^2)\cos\theta - dm^2R\sin\theta}{Id^2m\rho + d^2m^3\rho} \\ \frac{(I^2 + d^2m^2)\sin\theta - dm^2R\cos\theta}{Id^2m\rho + d^2m^3\rho} & \frac{(I^2 + d^2m^2)\sin\theta + dm^2R\cos\theta}{Id^2m\rho + d^2m^3\rho} \\ \frac{IR}{I^2\rho + d^2m^2\rho} & -\frac{IR}{I^2\rho + d^2m^2\rho} \end{bmatrix}$$

IV.The two loop control design of tracking target

The objective of this paper is to propose a controller such that the input torque applied to the two wheels can drive the mobile to follow the designated path or chase after another cart. For further details of the controller design, the mobile tracking problem needs to be defined first which described as the following. Given a reference cart velocity $\upsilon_r = \begin{bmatrix} v_r \\ w_r \end{bmatrix}$ find the control input such that the mobile can move as the same trajectory as the reference cart. The proposed controller design of tracking mission can be accomplished by employing two loop designs. The inner loop is nonlinear decoupling input and the outer loop is output injection of the error between the reference trajectory and the mobile trajectory which is depicted as the figure 2.

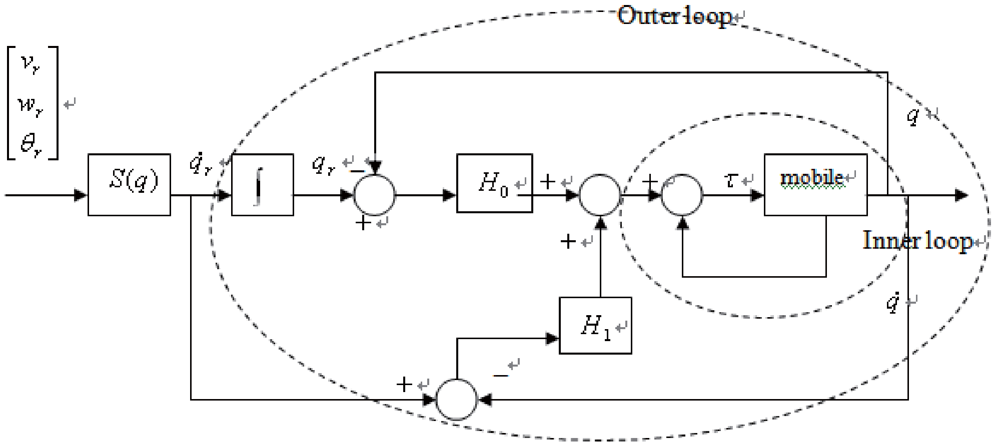

Figure 2. the two loop design control

Let the nonlinear decoupling input $\tau_{de} = \overline{C}(q)\dot{q} + \overline{A}(q)\lambda$ and outer loop injection $\tau_{inj} = H_0(q_r - q) + H_1(\dot{q}_r - \dot{q})$ plugging into (10), which is

$$\ddot{q} = \overline{M}^{+}(q)(\tau_{de} - \overline{C}(q)\dot{q} - \overline{A}(q)\lambda) + \tau_{inj} \quad (11)$$

$$= H_0(q_r - q) + H_1(\dot{q}_r - \dot{q}) \quad (12)$$

$$H_0 = k_0 I \ , \ H_1 = k_1 I$$

I is the identity matrix and k_0, k_1 are positive constants.

The proof the proposed controller can be stabilized system and follow the desired path is straightforward.

From (12), the closed loop system is derived as

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -H_0 & -H_1 \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ H_0 & H_1 \end{bmatrix} \begin{bmatrix} q_r \\ \dot{q}_r \end{bmatrix}$$

(13)

With adequately choosing k_0 and k_1, the closed system is asymptotically stable .In other words, the cart can follow the command $\begin{bmatrix} q_r \\ \dot{q}_r \end{bmatrix}$ which is the designated path.

V. Simulation

Given the reference input $V_r = \begin{bmatrix} v_r \\ w_r \end{bmatrix} = \begin{bmatrix} 2.5 \\ 1 \end{bmatrix}$ and the initial conditions was set to $\begin{bmatrix} x_0 \\ y_0 \\ \theta_0 \end{bmatrix} = \begin{bmatrix} 0.1 \\ 1 \\ 0 \end{bmatrix}$. Our goal is to design a controller such that mobile cart with the following property can follow the eference path: $m = 10$, $I = 5$, $R = 0.5$, $\rho = 0.05$, $d = 1$ and two constants are picked as $k_0 = 1000$, $k_1 = 100$. With the providing velocity $V_r = [2.5 \quad 1]^T$, the reference cart forms a circular path shown in figure 3.By utilizing the proposed controller, the outcome of simulation is presented in figure 4 which shows the mobile cart starts from the initial state then follow the circular path. From those results, it is shown feasible and effective of the proposed control law can accomplish the tacking mission.

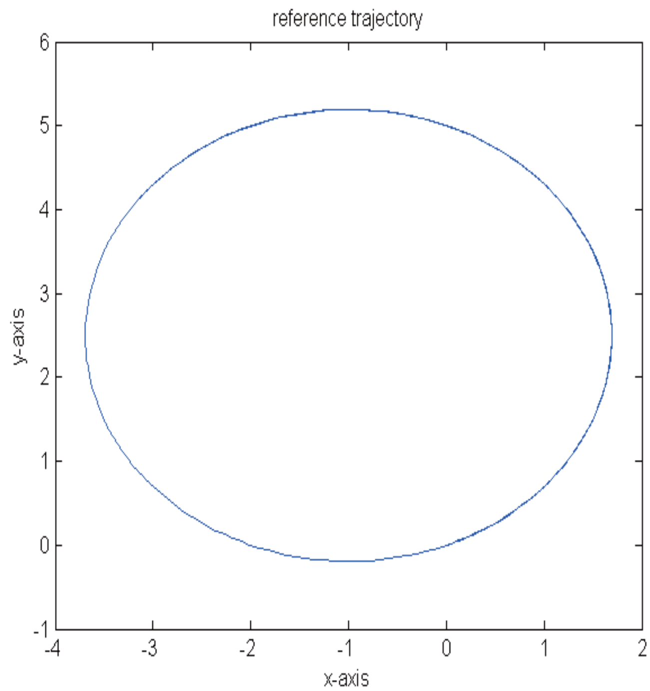

Fig 3. the reference trajectory

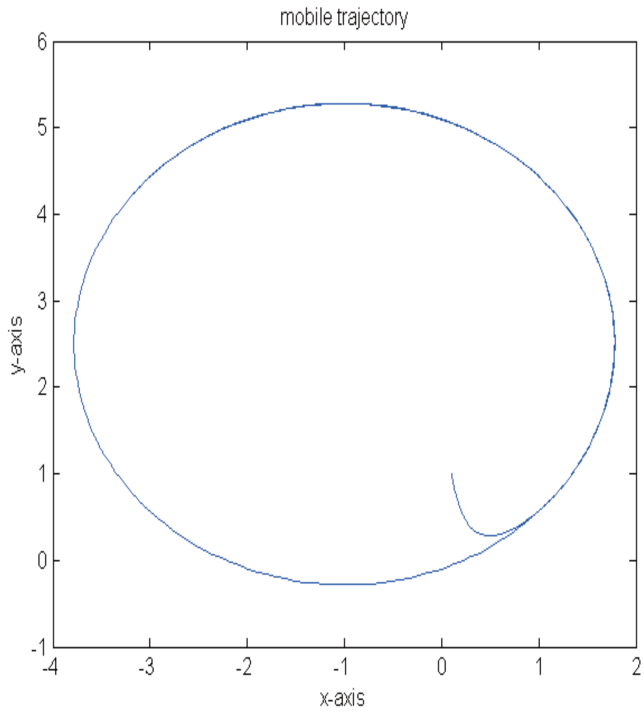

Fig4. the mobile trajectory

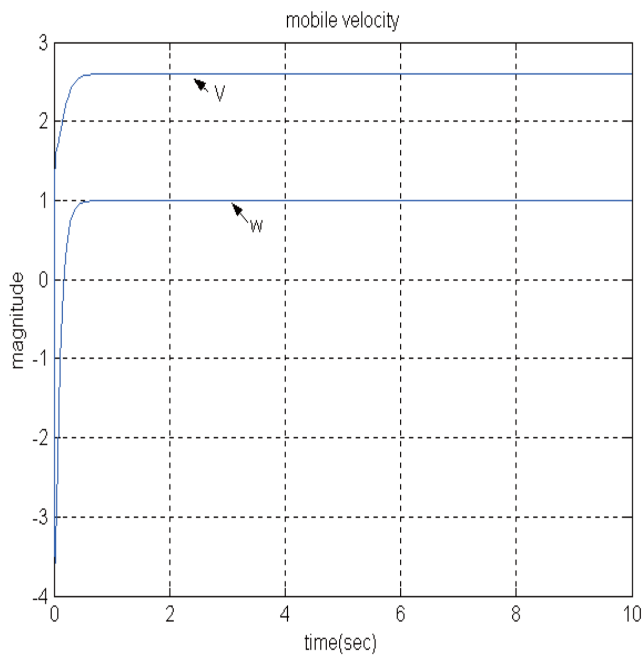

Fig 5. The mobile velocity

VI.The conclusion

In this paper, the study of control a mobile platform with Nonholonomic conditions by employing the nonlinear decoupling method is proposed. Due to the existence of the Jacobin matrix in the input term of the mobile cart dynamic equation, it is difficult to directly utilize the nonlinear decoupling technique. By providing a pseudo-inverse matrix the problem can be solved. From simulations, it is shown feasible and effective of the proposed control law can accomplish the tacking mission.

Reference

[1] J. Barraquand and J-C Latombe, "Nonholonomic Multibody Mobile Robots: Controllability and Motion Planning in the Presence of Obstacles," *Proc. IEEE International Conference on Robotics and Automation*, pp. 2328-2335, Sacramento, CA, April 1991.

[2] Yoshio Yamamoto and Xiaoping Yun," Coordinating Locomotion and Manipulation of a Mobile Manipulator" *Proceedings of the 31st IEEE Conference on Decision and Control*, pp.2643-2648, Tucson, Arizona, December 1992.

[3] Yoshio Yamamoto and Xiaoping Yun," Internal Dynamics of a Wheeled Mobile Robot" *Proceedings of the 1993 IEEE/RSJ International Conference on Intelligent Robots and Systems*,pp.1288-1294,Yokohama, Japan July 1993.

[4] Yoshio Yamamoto and Xiaoping Yun," Modeling and Compensation of the Dynamic Interaction of a Mobile Manipulator" *Robotics and Automation 1994 Proceedings 1994 IEEE International Conference*, pp.2187-2192.

[5] B. Steer," Trajectory Planning for a Mobile Robots" *The International Journal of Robotics Research,* 8(5): pp.3-14, October 1989.

[6] B. d'Andrea-Novel, G. Bastin, and G. Campion," Modeling and Control of non Holonomic Wheeled Mobile Robots" *Proceedings of 1991 International Conference on Robotics and Automation*, pp.1130-1135, Sacramento, CA, April 1991.

[7] R. Feierro and R. Lewis, " Control of a Nonholonomic Mobile Robot using Neural Networks" *IEEE Trans. Neural Networks,* vol.9, no.4, pp. 589-600. July, 1998.

[8] I. Kolmanovsky and

N.McClamroch,“ Development in Nonholonmic Control Problems” *IEEE Control Systems*, 20-26, December 1995.

[9] Sheng Lin and A.Goldenberg,” Robust Damping Control of Wheeled Mobile Robots” *Proceedings of the 2000 IEEE International Conference*, San Francisco, pp.2919-2924, CA, April 2000.

[10] Sheng Lin and A.Goldenberg,” Neural-Network Control of Mobile Manipulators” *IEEE Transactions on Neural Networks*, pp.1121-1133, Vol.12, No 5, September 2001.

[11] Y. Kanayama, Y. Kimura, F. Norguchi, “ A Stable Tracking Control Method for an Autonomous Mobile Robot,” *Proc. IEEE. International Conference on Robotics and Automation,* vol.1 pp.384-389, May 1990.

[12] C. Samson and K. Ait-Abderrahim, “Feedback Control of a Nonholonomic Wheeled Cart in Cartesian Space,” *Proc. IEEE International Conference on Robotics and Automation*, pp. 1136-1141, Sacramento, CA, April 1991.

[13] P. Morin and C. Samson, ”Trajectory tracking for non-holonomic vehicles: overview and case study,” *Fourth International Workshop on Robot motion and Control,* pp. 139-153. June 17-20, 2004

[14] R. Feierro and R. Lewis, “ Control of a Nonholonomic Mobile Robot: Backingsteping Kinematics into Dynmics” *Journal of Robotic System,* 14(3), pp.149-163., 1997.