The Design of Decoupling Predictive Controller for Multivariable System

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ABSTRACT

This paper presents a decoupling predictive controller for multivariable system. The stochastic decoupling polynomial matrix representation of the system is established. The decoupling control design is developed based on the minimization of a generalized predictive performance criterion. The set point tracking, disturbance rejection and robustness capabilities of the proposed method can be improved by appropriate adjustments of the tuning parameters in the criterion function. The proposed method can be less computational and more effective for the multivariable system. Through computer simulations, the proposed method has been shown to be powerful under set-point changes, load disturbances and significant plant uncertainties.

Key Words : Decoupling Predictive Control, Multivariable System

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I. INTRODUCTION

The multivariable control theories and techniques have been well documented in [1]-[7]. Although, those have been proved to have good performance and proper robustness, they have the time-consuming computational requirements. To overcome the shortcoming, the paper is expected to design a decoupling predictive controller for multivariable system. The proposed method will be shown to perform as well as the multivariable control approaches.

Decoupling predictive control design has received much significant attentions over past decades. A globally convergent direct adaptive decoupling predictive control algorithm was presented by Chai [8] with the assumption that the system interaction matrix is known diagonal. Qin *et al.* [9] designed a robust adaptive decoupling predictive design for generalized predictive control with neural network.

The objective of this paper is to combine adaptive control and decoupling predictive control for developing a less computational and more powerful adaptive predictive decoupling controller and to study its feasibility and effectiveness. The outline of this paper is organized as follow. The decoupling mathematical model is established in Section II. The decoupling generalized predictive control law is derived in Section III. Using computer simulation to study the ability of performance of the algorithm with a fixed, well-tuned, decoupling predictive controller is investigated in Section VI. Section V concludes the paper.

II. MATHEMATICAL MODEL

The process to be controlled is assumed to be an n-input n-output multivariable system described by the controlled autoregressive integrated moving average (CARIMA) model:

$$\overline{A}(z^{-1})y(k)=\overline{B}(z^{-1})u(k-d)+\varepsilon(k) \tag{1}$$

where

$$\overline{A}(z^{-1})=I-A_1z^{-1}-A_2z^{-1}-\cdots-A_{nA}z^{-nA}$$

$$\overline{B}(z^{-1})=B_0+B_1z^{-1}+B_2z^{-1}+\cdots+B_{nB}z^{-nB}$$

$y(k)$ and $u(k-d)$ denote n-vectors of multivariable system output and input, d is the discrete-time time delay. $\overline{A}(z^{-1})$ and $\overline{B}(z^{-1})$ are two $n\times n$ polynomial matrices. $\varepsilon(k)$ is an n-vector of mutually independent white Gaussian noises with zero mean.

To design the proposed decoupling predictive controller, we diagonalize the system as

$$A(z^{-1})y(k)=B(z^{-1})u(k-d)+N(k-1)+\varepsilon(k). \tag{2}$$

Note $A(z^{-1})$ and $B(z^{-1})$ are diagonal polynomial matrices, $N(k-1)$ can be though of as the model errors including unmodeled dynamics and coupling elements.

III. DECOUPLING PREDICTIVE CONTROL LAW

This section develops the decoupling predictive control for improving performance and robustness of the multivariable system. To derive the predictive control law, the following two equalities are used to solve for $E_j(z^{-1})$, $F_j(z^{-1})$, and $G_j(z^{-1})$, so as to find the j step-ahead predictor of $y(k)$:

$$I = \Delta E_j(z^{-1})A(z^{-1}) + z^{-j}F_j(z^{-1}) \tag{3}$$

$$E_j(z^{-1})B(z^{-1}) = G_j(z^{-1}) \tag{4}$$

where $j = 1,2,\cdots$, and

$$E_j(z^{-1}) = I + E_{j,1}z^{-1} + \cdots + E_{j,j-1}z^{-(j-1)}$$

$$F_j(z^{-1}) = F_{j,0} + F_{j,1}z^{-1} + \cdots + F_{j,nA}z^{-nA}$$

$$G_j(z^{-1}) = G_{j,0} + G_{j,1}z^{-1} + \cdots + G_{j,j+nB-1}z^{-(j+nB-1)}$$

After that, the j step-ahead predictive of $y(k)$ is derived by

$$\hat{y}(k+j) = F_j(z^{-1})y(k) + G_j(z^{-1})\Delta u(k+j-d) + E_j(z^{-1})\Delta N(k+j-1) \tag{5}$$

The generalized predictive control law is obtained, so as to minimize the expectation of the following quadratic cost function:

$$J = E\left\{ \sum_{j=N_1}^{N_2} \left\| \hat{y}(k+j) - W_j r(k+j) + H_j(z^{-1})\Delta N(k+j-1) \right\|^2 + \sum_{j=d}^{d+N_u-1} \left\| Q_j \Delta u(k+j-d) \right\|^2 \right\} \tag{6}$$

where $r(k)$ is an input reference signal and W_j is the feed forward gain. $H_j(z^{-1})$ is a diagonal polynomial matrix to be determined, whose role is to remove the effect of $N(k-1)$ on the closed-loop control system. Then, Q_j is a $n\times n$ selected weighting matrix, and $\Delta u(k)=(1-z^{-1})u(k)$. N_u, N_1 and N_2 denote the control horizon, the minimum output horizon, and the maximum output horizon, respectively. For the problem considered in the paper, N_1 must be chosen to be d, and N_2 is set to N_1+N-1, $N\geq 1$, where N denotes the prediction range.

The cost function (6) can be rewritten by

$$J=E\Big\{ (Fy(k)+\overline{G}U+L\Delta u(k-1)+\overline{M}N-WR)^T (Fy(k)+\overline{G}U+L\Delta u(k-1)+\overline{M}N-WR)+U^TQU\Big\} \tag{7}$$

where

$$F=\left[F_{N_1}(z^{-1}) \quad F_{N_1+1}(z^{-1}) \quad \cdots \quad F_{N_2}(z^{-1})\right]^T$$

$$\overline{G}=\begin{bmatrix} G_{N_1,0} & 0 & \cdots & 0 \\ G_{N_1+1,1} & G_{N_1+1,0} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ G_{N_2,N-1} & G_{N_2,N-2} & \cdots & G_{N_2,N-N_u} \end{bmatrix}.$$

$$U=\left[\Delta u(k)^T \quad \Delta u(k+1)^T \quad \cdots \quad \Delta u(k+N_u-1)^T\right]^T$$

$$L=\left[L_d(z^{-1}) \quad L_{d+1}(z^{-1}) \quad \cdots \quad L_{d+N_u-1}(z^{-1})\right]^T$$

$$N=\left[\Delta N(k+N_1-1)^T \quad \Delta N(k+N_1)^T \quad \cdots \quad \Delta N(k+N_2-1)^T\right]^T$$

$$\overline{M}=diag\left[E_d(z^{-1})+H_d(z^{-1}) \quad E_{d+1}(z^{-1})+H_{d+1}(z^{-1}) \quad \cdots \quad E_{d+N_u-1}(z^{-1})+H_{d+N_u-1}(z^{-1})\right]$$

$$W=\left[W_d \quad W_{d+1} \quad \cdots \quad W_{d+N_u-1}\right]$$

$$R = \left[r(k+N_1)^T \quad r(k+N_1+1)^T \quad \cdots \quad r(k+N_2)^T \right]^T$$

$$Q = diag\left[Q_d \quad Q_{d+1} \quad \cdots \quad Q_{d+N_u-1} \right].$$

Because the cost function J is quadratic in U, a minimum solution for U is easily found. Thus, it is required to find U such that

$$\frac{\partial J}{\partial U} = 0 \tag{8}$$

Therefore, the optimal predictive control U^* satisfies the following condition:

$$U^* = (\overline{G}^T \overline{G} + Q)^{-1} \overline{G}^T (WR - Fy(k) - L\Delta u(k-1) - \overline{M}N). \tag{9}$$

Generally speaking, for a large value of N_u, the computation for implementing generalized predictive control is time consuming. To reduce the computational load, we let the control horizon be one. Under the assumption, we have

$$G = \begin{bmatrix} G_{N_1} \\ G_{N_1+1} \\ \vdots \\ G_{N_2} \end{bmatrix} \equiv \begin{bmatrix} G_{N_1,0} \\ G_{N_1+1,1} \\ \vdots \\ G_{N_2,N-1} \end{bmatrix}$$

and

$$Q_0 = Q_d^T Q_d = diag\left[q_1 \quad q_2 \quad \cdots \quad q_n \right]$$

Then, Eq. (9) becomes

$$U^* = (G^T G + Q_0)^{-1} G^T (WR - Fy(k) - L\Delta u(k-1) - \overline{M}N). \tag{10}$$

Since the receding horizon control, we define Λ to be the first n rows of $(G^T G + Q_0)^T G^T$. By selecting the weighting polynomial matrix $H_j(z^{-1})$ so as to satisfy $(G^T G + Q_0)^T G^T \overline{M}N = M(z^{-1})\Delta N(k+d-1)$, where $M(z^{-1})$ is a diagonal polynomial matrix, yields

$$\Delta u(k) = \Lambda(WR - Fy(k) - L\Delta u(k-1)) - M\Delta N(k+d-1) \tag{11}$$

Substituting $\Delta u(k)$ into Eq. (2), the closed-loop system is

$$((I+\Lambda Lz^{-1})\Delta A(z^{-1})+B(z^{-1})z^{-d}\Lambda F)y(k)$$
$$=B(z^{-1})z^{-d}\Lambda WR+((I+\Lambda Lz^{-1}-B(z^{-1})M(z^{-1}))\Delta N(k-1) \qquad (12)$$

To realized closed-loop decoupling control, the term containing $\Delta N(k-1)$ must be zero. Thus, we have

$$I+\Lambda Lz^{-1}-B(z^{-1})M(z^{-1})=0 \qquad (13)$$

We obtain the control law (11) where $M(z^{-1})$ is determined by (13), such that $N(k-1)$ corresponding to the coupling and unmodelled components of the process can be completely removed from the closed-loop system. Using (12) and (13) so as to prove that the system output follows the reference signal properly, and W_j is chosen by

$$W[\underbrace{I \quad I \quad \cdots \quad I}_{N}]^T=F\big|_{z=1} \qquad (14)$$

VI. COMPUTER SIMULATIONS

The objective of this simulation example is to study the feasibility of the proposed decoupling predictive controller in controlling the heating process [3-5]. The simulation study also includes an investigation of the effect of load disturbance on the heating system employing the proposed controller, and implemented using MATLAB. The control algorithm is applied to the temperature control of a 4-input 4-output plastic injection molding process. This system can be use in decoupling discrete-time mathematical model as Eq. (2). From (11), the control signal $u_i(k)$ will be derived as follows

$$u_i(k) = u_i(k-1) + \sum_{j=N_1}^{N_2} \alpha_{ij} w_{ij} r_j(k+j) - \sum_{j=N_1}^{N_2} \alpha_{ij}(f_{ij,0} + f_{ij,1}z^{-1})y_i(k)$$

$$- \sum_{j=N_1}^{N_2} \alpha_{ij}(l_{ij,0} + l_{ij,1}z^{-1} + \cdots + l_{ij,d-2}z^{-(d-2)})\Delta u_i(k-1)$$

$$-\frac{1}{\hat{b}_i}(1 + \sum_{j=N_1}^{N_2} \alpha_{ij}(l_{ij,0}z^{-1} + l_{ij,1}z^{-2} + \cdots + l_{ij,d-2}z^{-(d-1)})\Delta n_i(k+d-1) \qquad (15)$$

where

$$\alpha_{ij} = \sum_{j=N_1}^{N_2} g_{ij} \big/ (g_{ij}^2 + q_i) \ , \quad w_{ij} = 1$$

$$g_{iN_1} = b_i \ , \quad g_{iN_1+1} = (1+a_i)b_i \ , \quad g_{iN_2} = (\sum_{m=0}^{N-1} a_i^m) b_i$$

$$f_{ij,0} = 1 + a_i + a_i^2 + \cdots + a_i^j \ , \quad f_{ij,1} = a_i + a_i^2 + \cdots + a_i^j$$

$$l_{ij,0} = \sum_{m=0}^{j-d+1} a_i^m \ , \quad l_{ij,1} = \sum_{m=0}^{j-d+2} a_i^m \ , \quad l_{ij,d-2} = \sum_{m=0}^{j-1} a_i^m$$

The simulation were performed for two sets of reference input $r(k)$, and the system parameters, the time delay, the predictive range and the weighting value given as follow

$$N = 20\ ,\quad d = 8$$

$$Q_0 = diag[0.02 \quad 0.02 \quad 0.02 \quad 0.01]$$

$$r(k) = \begin{cases} [200^oC \quad 210^oC \quad 220^oC \quad 230^oC], & 0 < k \le 500 \\ [250^oC \quad 260^oC \quad 270^oC \quad 280^oC], & 500 < k \le 1000 \end{cases}$$

$$A = diag[0.9851 \quad 0.9892 \quad 0.9891 \quad 0.9835]$$

$$B = diag[0.0017 \quad 0.0018 \quad 0.0017 \quad 0.0021]$$

$$N(k-1) = diag[1.0310 \quad 1.5282 \quad 1.0846 \quad 0.0695]$$

and the uncorrelated Gaussian noise with covariance 0.1 is added to the system. Fig. 1 and Fig.2 show the set-point tracking responses and control signals of four temperature zones in the heated barrel. It can be seen from the figure in which the performance is excellent.

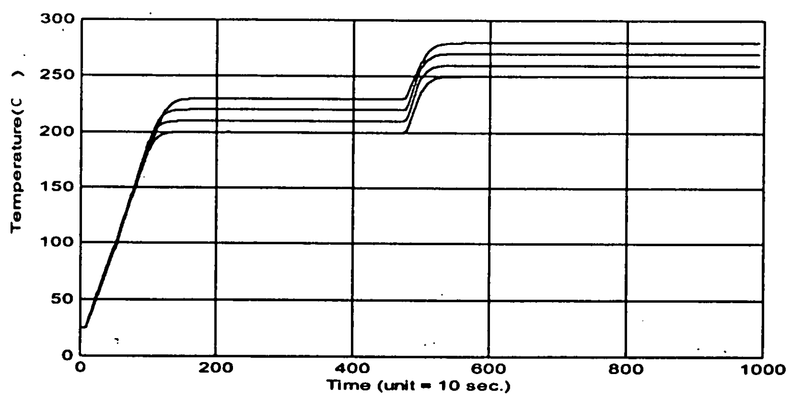

Fig. 1. Simulation result for set-point tracking responses of four temperature zones

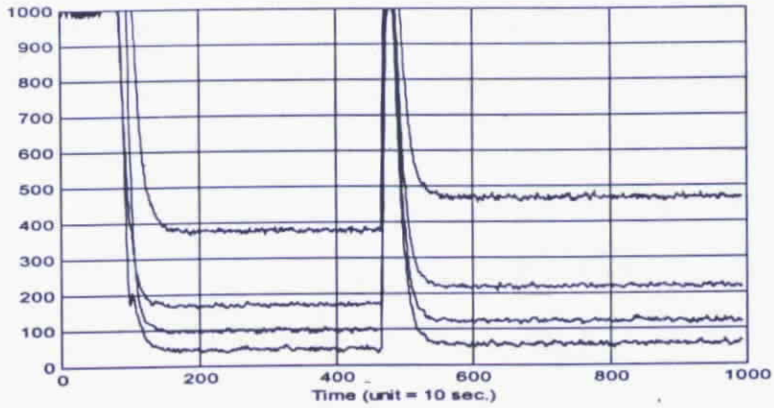

Fig. 2. Simulation result for control signals of four temperature zones

In order to investigate the relative effect of load disturbance in the performance of the proposed controller, the mathematical model was changed such that

$$y(k) = Ay(k-1) + Bu(k-d) + N(k-1) + v(k) + \varepsilon(k)$$

where

$$v(k) = [5.0 \quad 5.0 \quad 5.0 \quad 5.0]^T, \qquad \text{at } k = 300,$$

$$v(k) = [-5.0 \quad -5.0 \quad -5.0 \quad -5.0]^T, \quad \text{at } k = 750,$$

$$v(k) = [0.0 \quad 0.0 \quad 0.0 \quad 0.0]^T, \qquad \text{otherwise.}$$

Fig. 3 shows the simulation result of the decoupling predictive controlled, which is controlling the temperature of the heating process under load disturbance. The proposed controller has a desirable performance.

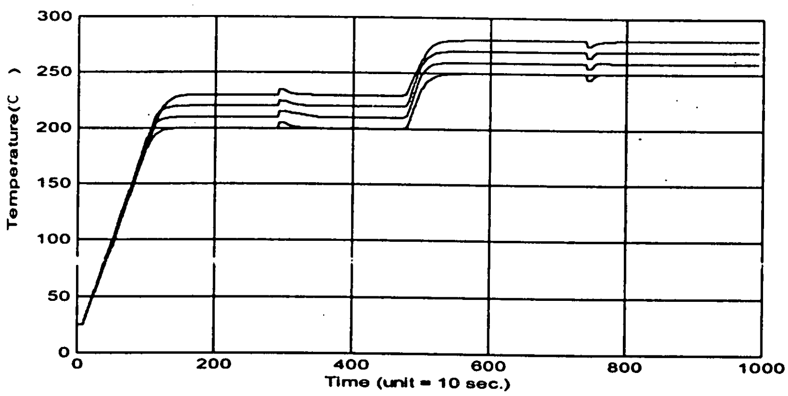

Fig. 3. Simulation result for decoupling predictive control in the presence load disturbance.

To test the robustness of the decoupling predictive controller, the third simulation was performed where the system parameters were artificially changed. The system parameters a_i and b_i with 10% variation were applied to the barrel after the 300*th* sample. Fig. 4 shows that the proposed controller could adapt to change in the plant parameters, and the control performance was satisfactory.

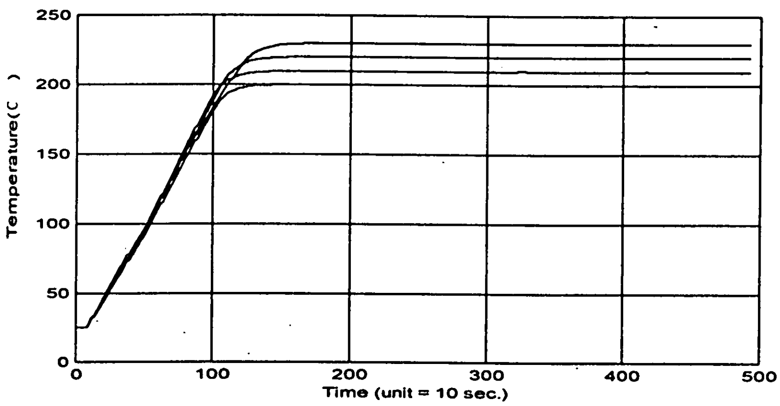

Fig. 4. Behavior of the decoupling predictive controller when the changed system parameters at $k \geq 300$.

V. CONCLUSIONS

This paper has presented a systematic design methodology for developing a decoupling predictive controller for multivariable system. The set-point tracking and disturbance rejection of the proposed method can be improved by appropriate adjustments of the tuning parameters in the criterion function and the design approach is proved to be less computational and more effective. Through computer simulations, the proposed method has been proved to powerful under set-point changes, load disturbances, and parameter variations.

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多變數系統之解耦預估控制器設計

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摘　要

本文提出多變數系統解耦預估控制器之設計技術。本研究首先建立系統的解耦離散隨機數學模型，接著在最小化廣義性能指標以發展解耦預估控制器，此設計具有良好的追蹤性能、消除干擾與參數變動下的強健能力，並且在控制多變數系統時有更省時的效率。由電腦模擬可證實本文所提方法在設定點追蹤與干擾影響下具有可行性與有效性。

關鍵詞：解耦預估控制、多變數系統

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