能量消散法之強健性控制器設計

楊伯華

摘要

本論文係針對具有外部輸入至控制輸出之直接前饋項問題，以能量耗散法設計strictly proper之控制器，文中對能量耗散控制器與知名之GD狀態空間控制器公式比較，並舉例說明。在某些情況下，strictly proper能量耗散控制器與non-strictly proper GD控制器可達同樣之H_∞性能。

關鍵字：控制、強韌控制、能量耗散法。

楊伯華：本校機械工程學系助理教授

STRICTLY PROPER LINEAR H_∞ CONTROLLER DESIGN BY ENERGY DISSIPATION APPROACH

Pao-Hwa Yang

Abstract

In this paper, a strictly proper H_∞ controller design based on the energy dissipation approach is proposed for a linear problem with direct feedthrough from an exogenous input to the controlled output. A comparison of the energy dissipation controller with the state-space formulas of Glover and Doyle (GD) is made. In some cases, the proposed strictly proper dissipative controller is as good as the non-strictly proper GD H_∞ controller in terms of H_∞ performance.

Keywords: H_∞ control, robust control, energy dissipation, strictly proper dissipative controller.

Pao-Hwa Yang: Assistant Professor, Department of Mechanical Engineering, HIT

1. INTRODUCTION

The energy dissipation approach, originally developed by Willems [1] and Hill and Moylan [2], has now drawn attention of many investigators and been successfully employed to the nonlinear H_∞ control problem [3-7]. In [5], by the energy dissipation approach and the separation principle, Ball, Helton, and Walker (BHW) derived a necessary and sufficient condition for the existence of solution to the problem and presented a formula of constructing a nonlinear H_∞ dissipative controller. In [5], for the reason of simplicity, BHW only considered a special case for the nonlinear H_∞ control problem with zero $D_{11}(X)$, which excludes the direct feedthrough term from the exogenous input to the controlled output. For the case with $D_{11}(X)\neq 0$, the equations involved in the construction of the nonlinear H_∞ dissipative controllers are much more complicated than those considered in [3-5]. The formulas of constructing nonlinear H_∞ controllers for the more general case are given in [8,9].

In this paper, we consider the design of strictly proper H_∞ controllers by the energy dissipation approach for the linear problem with nonzero feedthrough term (D_{11}). In fact this problem is just a linear version of that considered in [8,9]. Specifically, it is not our intention to derive the linear controller formulas in detail since they can be easily obtained by simplifying those in the nonlinear H_∞ dissipative controller [8,9]. Instead, we will concentrate on discussing the advantages and the limitations of the energy dissipation approach for the linear problem and comparing it with the well-known state-space H_∞ formulas of GD [10]. Hence, the information gathered from this study may also be of help in understanding more about the nonlinear dissipative controllers. Additionally, using the assumption that the two Riccati solutions are positive definite, both Isidori [4] and BHW [5] showed that the linear version of their controllers coincide with that of DGKF [9]. However, the linear version of the H_∞ dissipative controllers in general are different from that of GD [10]. It is due to the structure of the dissipative controller restricted to be strictly proper and the prescribed H_∞ upper bound assumed to be greater than the maximal singular value of D_{11}. Ultimately, we will show that when GD H_∞ controller is strictly proper, it is exactly the same as the dissipative H_∞ controller we propose. When the maximal singular value of D_{11} is less than the optimal H_∞ norm of the closed-loop system, the proposed strictly proper dissipative controller is as good as the non-strictly proper GD H_∞ controller in terms of H_∞ performance.

The rest of the paper is organized as follows. In Section 2, we briefly discuss some issues: the basic concept of the energy dissipation, the problem formulation, the Hamiltonian function of the closed-loop system, the assumptions, and the construction of a linear dissipative controller. In Section 3, we compare the linear H_∞ dissipative controller with the GD H_∞ controller. Furthermore, some illustrative examples are also enclosed to demonstrate the advantages and the limitations of the proposed controller. Section 4 gives the concluding remarks. Finally, in the Appendix, a proof is provided for some cases that GD H_∞ controller is identical to the dissipative H_∞ controller we propose.

2. Design of Dissipative Controllers

In this section, we will briefly introduce some concepts regarding to the dissipative system and employ them to construct a strictly proper H_∞ controller for a linear generalized plant with direct feedthrough from the exogenous input to the controlled output.

2.1. Concept of Dissipative System

Definition 2.1 *Consider the following system G*

$$G:\begin{cases}\dot{x}=F(x,w)\\ z=H(x,w)\end{cases} \qquad (1)$$

where w is the input and z is the output. With γ. a pre-assigned tolerance level, the system is said to be γ.-dissipative if there exists a nonnegative energy storage function E with E(x(0))=0 satisfying the following [9]

$$\int_0^T\left\{\|z\|^2-\gamma^2\|w\|^2\right\}dt\le E(x(0))-E(x(T))$$
$$=-E(x(T))\le 0 \qquad (2)$$

The inequality means that the H_∞ norm of the system is less than or equal to γ as T approaches to infinity. When γ=1, the inequality implies that the input energy is greater than or equal to the output energy. Accordingly, some energy has been dissipated and the system is called dissipative. From Definition 2.1, it is easy to see that the system is γ. -dissipative if and only if the energy Hamiltonian function

$$H=\|z\|^2-\gamma^2\|w\|^2+E_x\cdot F(x,w) \qquad (3)$$

is nonpositive in the domain of interest, where E_x denotes the derivative of E with respect to x.

2.2. Problem Formulation

Consider the following linear generalized plant:

$$G(s):\begin{cases}\dot{x}=Ax+B_1w+B_2u\\ z=C_1x+D_{11}w+D_{12}u\\ y=C_2x+D_{21}w\end{cases} \qquad (4)$$

where $x\in R^n$ is the state of the system, $z\in R^{p1}$ is the controlled output, $w\in R^{m1}$ is the exogenous input including all commands and disturbances, $u\in R^{m2}$ represents the control input, and $y\in R^{p2}$ is the measured output. The problem is to find a controller

$$K(s):\begin{cases}\dot{\xi}=A_K\xi+B_Ky\\ u=C_K\xi\end{cases} \qquad (5)$$

such that the closed-loop system is stable and γ-dissipative.

2.3. Hamiltonian Function

According to (3), the Hamiltonian function for the closed-loop system can be written as follows:

$$H_{A_K,B_K,C_K}(w,x,\xi)=\|z\|^2-\gamma^2\|w\|^2+E_\xi(x,\xi)\cdot$$
$$(A_K\xi+B_Ky)+E_x(x,\xi)(Ax+B_1w+B_2u] \qquad (6)$$

Now the problem is to find A_k, B_k and C_k such that the closed-loop system is stable and the Hamiltonian function is nonpositive for all w, x and ξ. Specifically, the problem is to find a nonnegative differentiable energy function $E(x,\xi)$ with $E(0,0) = 0$ so that there exist A_k, B_k and C_k such that

$$\max_{w} H_{A_K,B_K,C_K}(w,x,\xi) \le 0 \qquad (7)$$

2.4. Assumptions

The generalized plant described by (4) and the prescribed bound are assumed to satisfy the following:

(A1) (A,B_2) is stabilizable and (A,C_2) is detectable.

(A2) D_{12} is full column rank and D_{12} is full row rank. $D_\perp$ and $\tilde{D}_\perp$ are chosen such that $[D_\perp D_{12}]$ and $\begin{bmatrix} \tilde{D}_\perp \\ D_{21} \end{bmatrix}$ are unitary.

(A3) $rank\begin{bmatrix} j\omega I - A & B_{12} \\ C_1 & D_{12} \end{bmatrix} = n+m_2$ and $rank\begin{bmatrix} j\omega I - A & B_1 \\ C_2 & D_{21} \end{bmatrix} = n+p_2 \ \forall \omega \in$ R.

(A4) $(D_\perp C_1^T\text{-}A+B\hat{R}^{-1}\tilde{D}_{1.}^T C_1)$ is detectable.

(A5) $(\text{-}A+B_1 D_{.1}^T \tilde{R}^{-1} C, B_1 \tilde{D}_\perp^T)$ is stabilizable.

(A6) $\gamma > \sigma_{max}(D_{11})$, i.e., γ is greater than the maximum singular value of D_{11}, where

$$\hat{R} = D_{1\bullet}^T D_{1\bullet} - \begin{bmatrix} \gamma^2 I_{m1} & 0 \\ 0 & 0 \end{bmatrix} \qquad (8a)$$

$$\tilde{R} = D_{\bullet 1} D_{\bullet 1}^T - \begin{bmatrix} \gamma^2 I_{p1} & 0 \\ 0 & 0 \end{bmatrix} \qquad (8b)$$

$$D_{1\bullet} = [D_{11} \quad D_{12}] \qquad (8c)$$

$$D_{\bullet 1} = \begin{bmatrix} D_{11} \\ D_{21} \end{bmatrix} \qquad (8d)$$

Assumptions (A1) to (A3) are quite standard [10]. Assumption (A1) is the well-known necessary and sufficient condition for the existence of stabilizing controllers. (A2) is satisfied by most practical problems in which a weighted control input is part of z and a measurement noise is part of w. Assumption (A3) means that both $G_{12}(s) = C_1(sI - A)^{-1} B_2 + D_{12}$ and $G_{21}(s) = C_2(sI - A)^{-1} B_1 + D_{21}$ have no transmission zeros on the imaginary axis. Assumptions (A4), (A5), and (A6) are made to simplify the presentation. Later in this paper we will discuss how to remove (A4) and (A5). (A6) implies that $R := (D_{11}^T D_{11} - \gamma^2 I)^{-1}$. How to relax (A6) is still under investigation.

2.5. Construction of a -dissipative Controller

The condition for solution existence and the construction of a γ-dissipative controller are summarized in the following theorem.

Theorem 2.1 Consider the linear generalized plant defined by (4) which satisfies the assumptions in (A1) to (A6).

Let

$$R=(D_{11}^TD_{11}-\gamma^2I)^{-1} \quad (9a)$$

$$Q=(D_{21}RD_{21}^T)^{-1} \quad (9b)$$

$$M=(D_{12}^TD_{12}-D_{12}^TD_{11}RD_{11}^TD_{12})^{-1} \quad (9c)$$

and define

$$H_A=A-B_1RD_{11}^TC_1-(B_1RD_{11}^TD_{12}-B_2)\cdot MD_{12}^T(D_{11}RD_{11}^T-I)C_1 \quad (10a)$$

$$H_R=-(B_1RD_{11}^TD_{12}-B_2)M(D_{12}^TD_{11}RB_1^T-B_2^T)-B_1RB_1^T \quad (10b)$$

$$H_Q=C_1^T[I+(D_{11}RD_{11}^T-I)D_{12}MD_{12}^T]\cdot(D_{11}RD_{11}^T-I)C_1 \quad (10c)$$

$$J_A=A+B_1RD_{21}^TQ(-C_2+D_{21}RD_{11}^TC_1)-B_1RD_{11}^TC_1 \quad (10d)$$

$$J_R=B_1RD_{21}^TQD_{21}RB_1^T-B_1RB_1^T \quad (10e)$$

$$J_Q=-C_1^TC_1-(-C_2^T+C_1^TD_{11}RD_{21}^T)Q\cdot(-C_2+D_{21}RD_{11}^TC_1)+C_1^TD_{11}RD_{11}^TC_1 \quad (10f)$$

Then the closed loop system is dissipative if and only if there exist $X>0$, $Y_1>0$ such that

$$H_A^TX+XH_A+XH_RX-H_Q\le 0 \quad (11a)$$

$$J_A^TY_1+Y_1J_A+Y_1J_RY_1-J_Q\le 0 \quad (11b)$$

$$Y_1-X\ge 0 \quad (11c)$$

Furthermore, a γ-dissipative linear controller $K_{dis}(s)$ can be constructed by the following formulas

$$K_{dis}(s):=(A_K,B_K,C_K):=\left[\begin{array}{c|c}A_K & B_K\\ \hline C_K & 0\end{array}\right] \quad (12a)$$

$$B_K=-(Y_I-X)^{-1}(C_2^T-C_1^TD_{11}RD_{21}^T-Y_IB_1RD_{21}^T)Q \quad (12b)$$

$$C_K=M[D_{12}^T(D_{11}RD_{11}^T-I)C_1+(D_{12}^TD_{11}RB_1^T-B_2^T)X] \quad (12c)$$

$$A_K=A+B_2C_K-B_KC_2-(B_1-B_KD_{21})R[D_{11}^T(D_{12}^TC_K+C_1)+B_1^TX] \quad (12d)$$

Remarks

(i) If Assumptions (A4) and (A5) are satisfied, the solution of (11), X and Y_1, are always invertible and relate to the Riccati solutions X_∞ and Y_∞ in [10] as $X=X_\infty$, and $Y_1=\gamma^2Y_\infty^{-1}$. The coupling condition $Y_1-X\ \ge\ 0$ in (11c) corresponds to $\rho(Y_\infty X_\infty)\le\gamma^2$ in [10].

(ii) If Assumption (A4) is not satisfied, i.e., $(D_\perp^TC_1,-A+B\hat{R}^{-1}D_{1.}^TC_1):=(D_\perp^T$

C_1,-$\hat{A}$) is not detectable, one always can find an orthogonal similarity transformation matrix $U=[U_1 U_2]$ such that

$$U^T\hat{A}U=\begin{bmatrix} U_1^T\hat{A}U_1 & 0 \\ U_2^T\hat{A}U_1 & U_2^T\hat{A}U_2 \end{bmatrix} \quad (13a)$$

$$D_\perp^T C_1 U=\begin{bmatrix} D_\perp^T C_1 U_1 & 0 \end{bmatrix} \quad (13b)$$

which also decomposes the Riccati solution X_∞ as $\begin{bmatrix} X_1 & 0 \\ 0 & 0 \end{bmatrix}$ with $X_1 \geq 0$ and makes the subsystem ($U_1^T A U_1$, $U_1^T B_1$, $C_1 U$) satisfy (A4).

(iii) Similar remarks can be made for (A5) [12], and therefore (A4) and (A5) can be removed.

3. Comparison with GD Controller

In this section, we will compare the γ-dissipative controller $K_{dis}(s)$ with the well-known GD H$_\infty$ controller $K_{GD}(s)$ [10]. In the design of the γ-dissipative controller, if we restrict the structure of the controller to be strictly proper and assume γ is greater than the maximal singular value of D_{11}, denoted by $\gamma > \sigma_{\max}(D_{11})$, then the proposed dissipative controller is better than the GD controller. It is interesting to know the advantages and the limitation caused by these assumptions.

Let γ_{opt} be the optimal H$_\infty$ norm of the closed-loop system and be the generalized plant defined in (4), i.e.,

$$G(s) := \left[\begin{array}{c|c:c} A & B_1 & B_2 \\ \hline C_1 & D_{11} & D_{12} \\ \hdashline C_2 & D_{21} & D_{22} \end{array}\right] \quad (14)$$

which satisfies the assumptions (A1) to (A6). In addition, without loss of generality, we assume that $D_{11}=\begin{bmatrix} 0 \\ I \end{bmatrix}$, $D_{21}=[\,0 \;\; I\,]$and partition D_{11} as

$$D_{11} = \left[\begin{array}{c:c} D_{1111} & D_{1112} \\ \hdashline D_{1121} & D_{1122} \end{array}\right] \quad (15)$$

where $D_{1111} \in \mathrm{R}^{(p_1-m_2)\mathrm{x}(m_1-p_2)}$ and $D_{1122} \in \mathrm{R}^{m_2 \mathrm{x} p_2}$.

In the following, we will compare the γ-dissipative controller with the well- known GD H$_\infty$ controller in three cases.

Case 1: $D_{1122}=0$, and at least one of D_{1111}, D_{1112}, D_{1121} is zero.

The γ-dissipative controller $K_{dis}(s)=(A_k$, B_k, $C_k)$ is exactly the same as the GD controller $K_{GD}(s)$. Note that both controllers in this case are strictly proper. The proof is enclosed in the Appendix.

Case 2: The condition of Case 1 does not hold and $\gamma_{opt} > \sigma_{max}(D_{11})$.

The strictly proper γ-dissipative controller $K_{dis}(s)=(A_k, B_k, C_k)$ will make the H_∞ norm of the closed-loop system less than or equal to γ as the GD controller (which is not strictly proper) does.

Case 3: The condition of Case 1 does not hold and $\gamma_{opt} < \sigma_{max}(D_{11})$.

The non strictly proper GD controller may have a smaller closed-loop H_∞ norm than the strictly proper γ-dissipative controller. The limitation on the γ-dissipative controller is caused by the assumption that $\gamma > \sigma_{max}(D_{11})$.

Some examples are given in the following to illustrate the comparison of the GD and γ-dissipative controllers.

Example 1: Consider the following generalized plant

$$G(s)=\left[\begin{array}{ccc|ccc|cc}
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0\\
0 & -2 & 0 & 0 & 1 & 0 & 0 & 1\\
0 & 0 & -3 & 0 & 0 & 1 & 1 & 5\\
\hline
1 & 0 & 0 & 5.31 & 0 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0\\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1\\
\hline
2 & 1 & 0 & 0 & 1 & 0 & 0 & 0\\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0
\end{array}\right] \qquad (17)$$

D_{11} is partitioned as

$$D_{11}=\left[\begin{array}{c|c} D_{1111} & D_{1112}\\ \hline D_{1121} & D_{1122}\end{array}\right]=\left[\begin{array}{c|cc} 5.31 & 0 & 0\\ \hline 0 & 0 & 0\\ 0 & 0 & 0\end{array}\right]. \qquad (18)$$

which belongs to Case 1. A GD controller with γ = 8 can be obtained from the formulas in the proof (see Appendix), which is obtained as K_{GD} =

$$\left[\begin{array}{ccc|cc}
-6.2262 & -2.0135 & -0.0769 & 2.2973 & -0.5302\\
-0.0035 & -2.5294 & -0.9887 & 0.1827 & 0.2390\\
-1.3236 & -1.8682 & -8.0218 & 0.1494 & 0.9476\\
\hline
-2.8861 & -0.2164 & -0.0800 & 0 & 0\\
0.3719 & -0.1109 & -0.9884 & 0 & 0
\end{array}\right] \qquad (19)$$

From (12), the γ-dissipative controller is constructed exactly the same as the GD controller. The H_∞ norm of the closed-loop system is 6.6835 which is less than γ. The optimal H_∞ norm of the closed-loop system, γ_{opt}, can be obtained by iteratively reducing to the minimum at which the conditions of Theorem 2.1 still hold. For this problem, $\gamma_{\text{opt}} = 5.589581628$.

Example 2: Consider

$$G(s)=\left[\begin{array}{ccc|cccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 0.1 & 1\\ 0 & -2 & 0 & 0 & 1 & 0 & 0.4 & 1\\ 0 & 0 & -3 & 0 & 0 & 1 & 0 & 1\\ \hline 1 & 0 & 0 & 5.3 & 0 & 3 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 2 & 0 & 0\\ 0 & 0 & 1 & 0.2 & 0 & 0 & 0.2 & 0\\ 0 & 0.3 & 0.1 & 0.2 & 0.2 & 0.5 & 0.3 & 1\\ \hline 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{array}\right] \quad (21)$$

is partitioned as

$$D_{11}=\left[\begin{array}{ccc|c} 5.3 & 0 & 3 & 0\\ 0 & 0 & 2 & 0\\ 0.2 & 0 & 0 & 0.2\\ \hline 0.2 & 0.2 & 0.5 & 0.3 \end{array}\right] \quad (22)$$

which violates the conditions of Case 1. We found that $\sigma_{\max}(D_{11}) = 6.193914$ is smaller than the optimal H_∞ norm of the closed-loop system $\gamma_{\text{opt}} = 6.4949637$ and the problem belongs to Case 2. A GD controller with $\gamma = 6.5$ can be obtained as

$$K_{GD}=\left[\begin{array}{ccc|c} -417.53 & -426.64 & -427.36 & 425.39\\ 44.035 & 49.862 & 52.262 & 52.142\\ -69.166 & -65.735 & -68.722 & 65.338\\ \hline -6.3429 & -0.1051 & -0.1923 & -0.3035 \end{array}\right] \quad (23)$$

The H_∞ norm of the closed-loop system with the GD controller is 6.49999 which is less than γ. From (12), the γ-dissipative controller is constructed as

$$K_{dis}=\left[\begin{array}{ccc|c} -437.53 & -447.26 & -448.02 & 445.96\\ 44.166 & 52.057 & 54.462 & -54.333\\ -72.495 & -69.183 & -72.159 & 68.761\\ \hline -6.6380 & -0.4093 & -0.1125 & 0 \end{array}\right] \quad (24)$$

The H_∞ norm of the closed-loop system with the γ -dissipative controller is also 6.49999 which is less than γ. The optimal H_∞ norm of the closed-loop system, γ_{opt} , can be obtained by iteratively reducing γ to the minimum at which the conditions of Theorem 2.1 still hold. For this problem, $\gamma_{\text{opt}} = 6.4949637$. Note that the γ -dissipative controller is strictly proper while the GD controller has a direct feedthrough term.

Example 3: Consider

$$G(s)=\left[\begin{array}{ccc|ccc|c} 1 & 0 & 0 & 1 & 0 & 0 & 1\\ 0 & -2 & 0 & 0 & 1 & 0 & 1\\ 0 & 0 & -3 & 0 & 0 & 1 & 1\\ \hline 1 & 0 & 0 & 5.31 & 0 & 3 & 0\\ 0 & 1 & 0 & 0 & 0 & 2 & 0\\ 0 & 0 & 1 & 2 & 0 & 0 & 1\\ \hline 1 & 1 & 1 & 0 & 0 & 1 & 0 \end{array}\right] \quad (25)$$

is partitioned as

$$D_{11} = \left[\begin{array}{c:cc} 5.31 & 0 & 3 \\ 0 & 0 & 2 \\ \hdashline 2 & 0 & 0 \end{array}\right] \qquad (26)$$

which violates the conditions of Case 1. We will see that $\sigma_{max}(D_{11}) = 6.41841881$ is larger than the optimal H_∞ norm of the closed-loop system $H_\infty = 6.18443841$ and the problem belongs to Case 3. A GD controller with $\gamma. = 6.419$ can be obtained as

$$K_{GD} = \left[\begin{array}{ccc|c} -5.2342 & -1.5647 & -1.6970 & 1.0295 \\ -1.4906 & 0.3774 & 1.9684 & -2.5030 \\ -1.9209 & 1.7797 & -1.6526 & -1.8569 \\ \hline -1.5749 & 2.3081 & 1.9111 & -2.4494 \end{array}\right] \qquad (27)$$

The H_∞ norm of the closed-loop system with the GD controller is 6.24423 which is not only less than γ but also less than $\sigma_{max}(D_{11})$. From (12), the γ -dissipative controller is constructed as

$$K_{dis} = \left[\begin{array}{ccc|c} -12.226 & -6.6068 & -6.3613 & 5.3863 \\ -5.2554 & -2.3377 & -0.5433 & -0.1570 \\ -5.4660 & -0.7769 & -4.0177 & 0.3522 \\ \hline -5.5055 & -0.5265 & -0.7112 & 0 \end{array}\right] \qquad (28)$$

The H_∞ norm of the closed-loop system with the γ -dissipative controller is 6.41883 which is larger than that of the GD controller. In the γ -dissipation approach, γ cannot be smaller than $\sigma_{max}(D_{11}) = 6.41841881$; while in the GD approach, γ can be further reduced until it reaches its optimum $\gamma_{opt} = 6.18443841$ for this case 3 problem.

From this example, we know since the prescribed bound γ in GD controller can be further reduced to γ_{opt}, the non strictly proper GD controller may have a smaller closed-loop H_∞ norm than the strictly proper dissipative controller. The limitation of the proposed strictly proper dissipative controller is caused by the assumption $\gamma_{opt} > \sigma_{max}(D_{11})$.

4. Conclusions

In this paper, the design of strictly proper γ -dissipative controllers for the case with nonzero feedthrough term had been presented. We showed when GD H_∞ controller is strictly proper, it is exactly the same as the proposed dissipative controller. When the maximal singular value of D_{11} is less than the optimal H_∞

norm of the closed-loop system, the proposed strictly proper dissipative controller can achieve whatever H_∞ performance the non-strictly proper GD H_∞ controller can reach.

5. List of Notations

K_{dis}	the dissipative controller
K_{GD}	the Glover and Doyle controller
$R^n K$	n-dimensional Euclidean space
R^{mxn}	the set of real *mxn* matrices
$\|x\|^2$	the squared Euclidean norm equaling $x^T x$
γ_{opt}	the optimal norm of the closed-loop system
$\sigma_{max}(A)$	the maximal singular value of a matrix A
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	$:= C(sI-A)^{-1}B+D = G(s)$ G(s) has a state space representation (A,B,C,D)

6. Appendix

In this appendix, we will show for Case 1, indicated in Section 3, the linear dissipative controller is identical to the GD controller. Before proving, the following lemma is needed.

Lemma A.1: If T^{-1} exists, then

$$\begin{bmatrix} T & U \\ V & W \end{bmatrix} = \begin{bmatrix} T^{-1}+e\Delta^{-1}f & -e\Delta^{-1} \\ -\Delta^{-1}f & \Delta^{-1} \end{bmatrix} \quad \text{(a-1)}$$

where, $\boldsymbol{\Delta} = W - VT^{-1}U$, $e = T^{-1}U$ and $f = VT^{-1}$.

Proof:

The GD controller in [10] is given as

$$K_{GD}(s) = \left[\begin{array}{c|c} \hat{A} & \hat{B} \\ \hline \hat{C} & \hat{D} \end{array}\right] \quad \text{(a-2)}$$

where

$$\hat{A} = A + BF - \hat{B}\hat{D}_{21}^{-1}\hat{D}_{21}(C_2 + F_{12}) \quad \text{(a-3a)}$$

$$\hat{B} = -Z_\infty^{-1}L_2 + Z_\infty^{-1}(B_2 + L_{12})\hat{D}_{12}\hat{D}_{12}^{-1}\hat{D} \quad \text{(a-3b)}$$

$$Z_\infty = I - \gamma^{-2}Y_\infty X_\infty \quad \text{(a-3c)}$$

$$\hat{C} = F_2 - \hat{D}\hat{D}_{21}^{-1}\hat{D}_{21}(C_2 + F_{12}) \quad \text{(a-3d)}$$

$$\hat{D} = -D_{1121}D_{1111}^T(\gamma^2 I - D_{1111}D_{1111}^T)^{-1} D_{1112} - D_{1122} \quad \text{(a-3e)}$$

$\hat{D}_{12} \in \mathbf{R}^{m2 \times m2}$ and $\hat{D}_{21} \in \mathbf{R}^{p2 \times p2}$ are matrices satisfying

$$\hat{D}_{12}\hat{D}_{12}^T = I - D_{1121}(\gamma^2 I - D_{1111}^T D_{1111})^{-1} D_{1112} \quad \text{(a-4a)}$$

$$\hat{D}_{21}^T\hat{D}_{21} = I - D_{1122}^T(\gamma^2 I - D_{1111}D_{1111}^T)^{-1} D_{1112} \quad \text{(a-4b)}$$

$$F = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = -\hat{R}^{-1}[D_{1\bullet}^T C_1 + B^T X_\infty] \quad \text{(a-4c)}$$

$$L = [L_1 \quad L_2] = -[B_1 D_{\bullet 1}^T + Y_\infty C^T]\tilde{R}^{-1} \quad \text{(a-4d)}$$

$$\left[\begin{array}{c|c} & F^T \\ \hline L^T & D \end{array}\right] = \left[\begin{array}{c|ccc} & F_{11}^T & F_{12}^T & F_2^T \\ \hline L_{11}^T & D_{1111} & D_{1112} & 0 \\ L_{12}^T & D_{1121} & D_{1122} & I \\ L_2^T & 0 & I & 0 \end{array}\right] \quad \text{(a-4e)}$$

where $\hat{R}$, $\tilde{R}$, $D_{1\bullet}$, $D_{\bullet 1}$ are defined in (8).

Applying the condition of Case 1 to (a-3e), we have $\hat{D}=0$ and the GD H_∞ controller can be simplified as:

$$\hat{A} = A + BF - \hat{B}\hat{D}_{21}^{-1}\hat{D}_{21}(C_2 + F_{12}) \quad \text{(a-5a)}$$

$$\hat{B} = -Z_\infty^{-1}L_2 \quad \text{(a-5b)}$$

$$\hat{C} = F_2 \quad \text{(a-5c)}$$

Lemma A.1 can be used to expand $\hat{R}^{-1}$ and $\tilde{R}^{-1}$ which are defined in (8). Now,

$$\hat{R}^{-1} = \begin{bmatrix} R + R(D_{11}^T D_{12})M(D_{12}^T D_{11})R & -RD_{11}^T D_{12}M \\ -MD_{12}^T D_{11}R & M \end{bmatrix} \quad \text{(a-6)}$$

$$\tilde{R}^{-1} = \begin{bmatrix} R^T - R^T D_{11} D_{21}^T(\gamma^{-2}Q)D_{21}D_{11}^T R^T & R^T D_{11} D_{21}(\gamma^{-2}Q) \\ (\gamma^{-2}Q)D_{21}D_{11}^T R^T & \gamma^{-2}Q \end{bmatrix}$$

where R, M, Q were defined in (9).

Plugging (a-6) into (a-4c~e), we have the following:

$$F_1 = -R[D_{11}^T(D_{12}^T C_K + C_1) + B_1^T X_\infty \quad \text{(a-7a)}$$

$$F_2 = M[(D_{12}^T D_{11}RB_1^T - B_2^T)X_\infty + D_{12}^T(D_{11}RD_{11}^T - I)C_1] \quad \text{(a-7b)}$$

$$L_1 = -(B_1 D_{11}^T + Y_\infty C_1^T)[R^T - R^T D_{11} D_{21}^T(\gamma^{-2}Q)\cdot D_{21}D_{11}^T R^T] - (B_1 D_{21}^T + Y_\infty C_2^T)(\gamma^{-2}Q)D_{21}D_{11}^T R^T \quad \text{(a-7c)}$$

$$L_2 = [Y_I^{-1}(C_2^T - C_1^T D_{11}RD_{21}^T) - B_1 RD_{21}^T]Q \quad \text{(a-7d)}$$

where

$$Y_I = \gamma^2 Y_\infty^{-1} \quad \text{(a-7e)}$$

From (a-4d), we can see that

$$[0 \quad I]\begin{bmatrix} F_{11} \\ F_{12} \end{bmatrix} = F_{12}, \quad \text{i.e.,} \quad D_{21}F_1 = F_{12} \quad \text{(a-7f)}$$

Substituting the expressions from (a-7) into (a-5), we have the following

$$\hat{B} = -Z_\infty^{-1}L_2 = -(I - Y_I^{-1}X_\infty)^{-1}[Y_I^{-1}(C_2^T - C_1^T D_{11}RD_{21}^T) - B_1 RD_{21}^T]Q = B_K \quad \text{(a-8a)}$$

$$\hat{C} = F_2 = M[D_{12}^T(D_{11}RD_{11}^T - I)C_1 + (D_{12}^T D_{11}RB_1^T - B_2^T)X_\infty] = C_K \quad \text{(a-8b)}$$

$$\begin{aligned} \hat{A} &= A + BF - \hat{B}\hat{D}_{21}^T\hat{D}_{21}(C_2 + F_{12}) \\ &= A + B_1F_1 + B_2F_2 + B_K\hat{D}_{21}^{-1}[-\hat{D}_{21}(C_2 + F_{12})] \\ &= A + B_2C_K - B_1R[D_{11}^T(D_{12}C_K + C_1) + B_1^T X_\infty] \\ &\quad + B_K\{-C_2 + D_{21}R[D_{11}^T(D_{12}C_K + C_1) + B_1^T X_\infty]\} \\ &= A_k \end{aligned} \quad \text{(a-8c)}$$

where $K_{dis} := (A_k,\ B_k,\ C_k)$ is the γ-dissipative controller given in (12). Thus, the proof is completed.

7. References

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