Transient Response of Sudden Heating Laminated Magnetostrictive Shells

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Abstract

The transient responses of displacements and stresses of three-layer cross ply laminated magnetostrictive shells under rapid heating were obtained by using GDQ computational method. The dynamic equilibrium differential equations were normalized and discretized into the corresponding dynamic discretized equations with the GDQ method. The laminated magnetostrictive shell was considered with velocity feedback controlled gain, the sudden uniform heat input over its lower surface only and temperature load is linear of axial coordinate. With suitable control gain value in laminated magnetostrictive shell can reduce the amplitude of displacement to smaller value. The present solutions might be applied to the fields of mechanical and aerospace.

Keywords: transient responses, GDQ, magnetostrictive shell, rapid heating, controlled gain.

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快速加熱層疊磁縮板殼之暫態反應

洪志強

摘 要

以一般化微分級數(GDQ)電腦計算方法來做 3 層正交層疊磁縮板殼受到快速加熱作用下的位移和應力之暫態反應。動態平衡微分方程式用 GDQ 法來做正規化及離散化，得到相當的動態離散方程式。該磁縮板殼有考慮速度回授控制之影響，及線性均勻的快速加熱在內層板殼表面上。使用適當的控制增益值可以減小該層疊磁縮板殼之位移振幅。這項研究分析將可應用到機械和航太工程領域方面。

關鍵詞：暫態反應、一般化微分級數、磁縮板殼、快速加熱、控制增益值。

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Introduction

Magnetostrictive Terfenol-D ($Tb_{0.3}D_{0.7}Fe_{1.9}$) material can be applied to the design of sensors and actuators for making the function of faster response. Lee used Laplace transform and finite difference methods to calculate the transient values of thermal displacement and stress in the magnetothermoelastic multilayered conical shells [1]. Lee et al. [2] made the nonlinear analyses in the laminated composite shells with actuating magnetostrictive layers by using the Finite Element Method (FEM). Pradhan used the approach of FEM to calculate the vibration suppression of functionally graded material (FGM) shells with embedded magnetostrictive layers [3]. Kumar et al. made the FEM computation for the active control of cylindrical shell with magnetostrictive layer [4].

In 2009, Hong presented the rapid heating induced vibration analysis of laminated shell by using the GDQ method [5]. The time responses of axial, circumferential and normal displacements are obtained. Recently, there are seldom studies about the thermal vibration of rapid heating laminated magnetostrictive shell in open literature or commercial software. In 2007, Hong made the study about thermal vibration of magnetostrictive material in laminated plates with the GDQ method. The time responses of center displacement and stresses with and without velocity control have been obtained, respectively [6]. In 2005, Hong et al. presented the GDQ computation for the thermally induced vibration of a thermal sleeve [7]. In this paper, we use GDQ method to study the transient responses of a laminated magnetostrictive shell with two edges clamped condition under rapid heating loads. The expected contribution of this field: with velocity feedback control and with suitable control gain values in laminated magnetostrictive shell to reduce the amplitudes of displacement and stress into smaller value, respectively.

2. Dynamic Equilibrium Differential Equation

We consider a thin generally orthotropic multi-layered shells as shown in **Fig. 1**, the thermo-elastic stress-strain relationship of the k^{th} layer including thermal strain and magnetostrictive coupling effect can be given in the following equations [2, 6].

$$\begin{Bmatrix} \sigma_x \\ \sigma_\theta \\ \sigma_{x\theta} \end{Bmatrix}_{(k)} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_{(k)}$$

$$
\begin{Bmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_\theta - \alpha_\theta \Delta T \\ \varepsilon_{x\theta} - \alpha_{x\theta} \Delta T \end{Bmatrix}_{(k)} - \begin{bmatrix} 0 & 0 & \bar{e}_{31} \\ 0 & 0 & \bar{e}_{32} \\ 0 & 0 & \bar{e}_{36} \end{bmatrix}_{(k)} \begin{Bmatrix} 0 \\ 0 \\ \widetilde{H}_z \end{Bmatrix} \qquad (1)
$$

where α_x and α_θ are the coefficients of thermal expansion, $\alpha_{x\theta}$ is the coefficient of thermal shear, ΔT is the temperature difference between the laminate and curing area, $\overline{Q}_{ij}$ is the transformed reduced stiffness, $\varepsilon_x, \varepsilon_\theta, \varepsilon_{x\theta}$ are strains in terms of displacement components, respectively, $\bar{e}_{ij}$ are the transformed magnetostrictive coupling moduli, and $\widetilde{H}_z$ is the magnetic field intensity.

We can obtain the equilibrium differential equations in terms of displacement components u, v and w in the directions of axial x, circumferential θ and normal z, respectively.

$$
\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix} \qquad (2)
$$

Where L_{ij} can be represented in the partial derivatives of x, θ and t, with coefficient related to $A_{ij}, B_{ij}, D_{ij}, \rho_t$ and R as follows.

$$L_{11} = A_{11}\frac{\partial^2}{\partial x^2} + \frac{A_{66}}{R^2}\frac{\partial^2}{\partial \theta^2} - \rho_t \frac{\partial^2}{\partial t^2}$$

$$L_{12} = (\frac{B_{12}}{R^2} + \frac{A_{12}}{R} + \frac{A_{66}}{R} + 2\frac{B_{66}}{R^2})\frac{\partial^2}{\partial x \partial \theta}$$

$$L_{13} = \frac{A_{12}}{R}\frac{\partial}{\partial x} - B_{11}\frac{\partial^3}{\partial x^3} - (\frac{B_{12}}{R^2} + 2\frac{B_{66}}{R^2})\frac{\partial^3}{\partial x \partial \theta^2}$$

$$L_{21} = (\frac{B_{12}}{R^2} + \frac{A_{12}}{R} + \frac{A_{66}}{R} + \frac{B_{66}}{R^2})\frac{\partial^2}{\partial x \partial \theta}$$

$$L_{22} = (A_{66} + \frac{3B_{66}}{R} + \frac{2D_{66}}{R^2})\frac{\partial^2}{\partial x^2} + (\frac{2B_{22}}{R^3} + \frac{A_{22}}{R^2} + \frac{D_{22}}{R^4})\frac{\partial^2}{\partial \theta^2} - \rho_t \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial x}(N_a \frac{\partial}{\partial x})$$

$$L_{23} = (\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3})\frac{\partial}{\partial \theta} - (\frac{2B_{66}}{R} + \frac{B_{12}}{R} + \frac{2D_{66}}{R^2} + \frac{D_{12}}{R^2})\frac{\partial^3}{\partial x^2 \partial \theta} - (\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4})\frac{\partial^3}{\partial \theta^3}$$

$$L_{31} = -\frac{A_{12}}{R}\frac{\partial}{\partial x} + B_{11}\frac{\partial^3}{\partial x^3} + (\frac{B_{12}}{R^2} + 2\frac{B_{66}}{R^2})\frac{\partial^3}{\partial x \partial \theta^2}$$

$$L_{32} = -(\frac{A_{22}}{R^2} + \frac{B_{22}}{R^3})\frac{\partial}{\partial \theta} + (\frac{2B_{66}}{R} + \frac{B_{12}}{R} + \frac{4D_{66}}{R^2} + \frac{D_{12}}{R^2})\frac{\partial^3}{\partial x^2 \partial \theta} + (\frac{B_{22}}{R^3} + \frac{D_{22}}{R^4})\frac{\partial^3}{\partial \theta^3}$$

$$L_{33} = \frac{2B_{12}}{R} - D_{11}\frac{\partial^4}{\partial x^4} - (\frac{2D_{12}}{R^2} + \frac{4D_{66}}{R^2})\frac{\partial^4}{\partial x^2 \partial \theta^2} - \frac{D_{22}}{R^4}\frac{\partial^4}{\partial \theta^4} + \frac{2B_{22}}{R^3}\frac{\partial^2}{\partial \theta^2} - \frac{A_{22}}{R^2} - \rho_t \frac{\partial^2}{\partial t^2} + \frac{\partial}{\partial x}(N_a \frac{\partial}{\partial x})$$

$$b_1 = \frac{\partial \overline{N}_x}{\partial x} + \frac{1}{R}\frac{\partial \overline{N}_{x\theta}}{\partial \theta} + \frac{\partial \widetilde{N}_x}{\partial x} + \frac{1}{R}\frac{\partial \widetilde{N}_{x\theta}}{\partial \theta}$$

$$b_2 = \frac{\partial \overline{N}_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial \overline{N}_\theta}{\partial \theta} + \frac{1}{R}\frac{\partial \overline{M}_{x\theta}}{\partial x} + \frac{1}{R^2}\frac{\partial \overline{M}_\theta}{\partial \theta} + \frac{\partial \widetilde{N}_{x\theta}}{\partial x} + \frac{1}{R}\frac{\partial \widetilde{N}_\theta}{\partial \theta} + \frac{1}{R}\frac{\partial \widetilde{M}_{x\theta}}{\partial x} + \frac{1}{R^2}\frac{\partial \widetilde{M}_\theta}{\partial \theta}$$

$$b_3 = \frac{\partial^2 \overline{M}_x}{\partial x^2} + \frac{2}{R}\frac{\partial^2 \overline{M}_{x\theta}}{\partial x \partial \theta} + \frac{1}{R^2}\frac{\partial^2 \overline{M}_\theta}{\partial \theta^2} - \frac{\overline{N}_\theta}{R} + \frac{\partial^2 \widetilde{M}_x}{\partial x^2} + \frac{2}{R}\frac{\partial^2 \widetilde{M}_{x\theta}}{\partial x \partial \theta} + \frac{1}{R^2}\frac{\partial^2 \widetilde{M}_\theta}{\partial \theta^2} - \frac{\widetilde{N}_\theta}{R}$$

$\rho_t = \int_{-h/2}^{h/2} \rho dz$ in which ρ is the density, h is the thickness of shell, R is the mean radius, N_a is the pulsating axial load.

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} \overline{Q}_{ij} (1, z, z^2) dz$$

$$(\overline{N}_x, \overline{M}_x) = \int_{-h/2}^{h/2} (\overline{Q}_{11}\alpha_x + \overline{Q}_{12}\alpha_\theta + \overline{Q}_{16}\alpha_{x\theta})\Delta T(1,z) dz$$

$$(\overline{N}_\theta, \overline{M}_\theta) = \int_{-h/2}^{h/2} (\overline{Q}_{12}\alpha_x + \overline{Q}_{22}\alpha_\theta + \overline{Q}_{26}\alpha_{x\theta})\Delta T(1,z) dz$$

$$(\overline{N}_{x\theta}, \overline{M}_{x\theta}) = \int_{-h/2}^{h/2} (\overline{Q}_{16}\alpha_x + \overline{Q}_{26}\alpha_\theta + \overline{Q}_{66}\alpha_{x\theta})\Delta T(1,z) dz$$

$$\begin{Bmatrix} \widetilde{N}_x \\ \widetilde{N}_\theta \\ \widetilde{N}_{x\theta} \end{Bmatrix} = \sum_{k=1}^{N_m} \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \overline{e}_{31} \\ \overline{e}_{32} \\ \overline{e}_{36} \end{Bmatrix}^{(k)} \widetilde{H}_z dz$$

$$\begin{Bmatrix} \widetilde{M}_x \\ \widetilde{M}_\theta \\ \widetilde{M} \end{Bmatrix} = \sum_{k=1}^{N_m} \int_{z_k}^{z_{k+1}} \begin{Bmatrix} \overline{e}_{31} \\ \overline{e}_{32} \\ \overline{e}_{36} \end{Bmatrix}^{(k)} \widetilde{H}_z z^2 dz$$

Where N_m represents the layer number of magnetostrictive material.

3. Dynamic Discretized Equations

We consider the following the vibration behavior of displacement:

$$u = U(x)\cos(n\theta + \omega t)$$
$$v = V(x)\sin(n\theta + \omega t) \qquad (3)$$
$$w = W(x)\cos(n\theta + \omega t)$$

where ω (rad/sec) is the natural circular frequency and n is an integer for the circumferential wave number of the multilayered shell.

The following non-dimensional parameters are introduced:

$$X = x/L, \quad Z = z/h, \quad U = U(x)/L$$
$$V = V(x)/R, \quad W = W(x)/h \qquad (4)$$

where L is the length of shell.

For two edges were clamped, symmetric ($B_{ij} = 0$), orthotropic ($A_{16} = A_{26} = 0$, $D_{16} = D_{26} = 0$, $\alpha_{x\theta} = 0$) of laminated shell under temperature loading, we applied one-dimensional GDQ method to discretize the equilibrium differential equation (2), thus we could obtain the dynamic discretized equations and frequency parameter $f^* = \omega R \sqrt{\rho_t / A_{11}}$.

4. Computational Results

We consider the total three-layer $(0^{\circ m}/90^{\circ}/0^{\circ})$ cross-ply laminated shell. The superscript of m in $(0^{\circ m}/90^{\circ}/0^{\circ})$ denotes magnetostrictive material. The magnetostrictive Terfenol-D coupling modulus equation is $e_{31} = e_{32} = E^m d^m$ with $E^m = 26.5\,GPa$, $d^m = 1.67\times10^{-8}\,mA^{-1}$. The material properties of the typical host material and Terfenol-D are listed in **Table 1**.

Table 1 Properties of typical host and Terfenol-D

Properties	Typical host		Terfenol-D
	Inner	Outer	
E_1/E_2	25	40	1
G_{12}/E_2	0.5	0.6	$\frac{13.25}{26.5}$
ν_{12}	0.15	0.27	0.0
$\rho(lb/in^3)$	0.087	0.283	0.334179
$\alpha_x(1/{}^\circ F)$	6×10^{-6}	6.5×10^{-6}	12×10^{-6}
$\alpha_\theta(1/{}^\circ F)$	6×10^{-6}	6.5×10^{-6}	12×10^{-6}

The upper surface and all edges of the shell are considered to be thermally insulated. The temperature T_0 depends only on Z and t, where $Z = z/h$. $T_0 = T_0(Z,t)$ can be found as in the following equation [8, 9]:

$$T_0 = \frac{hq_0}{\kappa}[\frac{\beta t}{\pi^2} + \frac{1}{2}(\frac{z}{h}+\frac{1}{2})^2 - \frac{1}{6} - \frac{2}{\pi^2}\sum_{j=1}^{\infty}\frac{(-1)^j}{j^2}e^{-j^2\beta t}\cos j\pi(\frac{z}{h}+\frac{1}{2})] \qquad (5)$$

Where $\beta = \pi^2\kappa/h^2$, κ is the coefficient of thermal conductivity, q_0 is the heat flux.

For simplification, we considering the vibration case for the conditions: $\overline{M}_x = \overline{M}_\theta = \overline{M}_{x\theta} = 0$, $\overline{N}_x = -N_a x$, $\Delta T = \frac{A_{11}}{R^2}T_0 x$.

The temperature $T_0 = T_0(\frac{1}{2},t)$ is applied in the sudden uniform input over the lower surface $Z = \frac{1}{2}$ of inner layer only.

We firstly to investigate the dynamic convergence of the frequency parameter f^* and center displacement $W(L/2)$ with $R/h = 500$, $L/R = 10$, circumferential wave number $n = 4$, $\theta = 1$ radian, time $t = 1$ sec and without velocity feedback control gain value $k_c c(t) = 0$ under $T_0 = T_0(\frac{1}{2},t)$ and $q_0 = 2\,Btu/\sec\cdot in^2$ with clamped-clamped boundary condition. We find that the N = 73 grid point has the good convergence results and can be used further in the GDQ calculation of transient responses for deflection and stress.

Fig. 2 shows that transient responses of dominant normal center displacement $W(L/2)$ under sudden uniform input: $T_0(\frac{1}{2},t)$, $q_0 = 2\,{}^{Btu}\!/_{\sec\cdot in^2}$, $\theta = 1$ radian, N = 73, for $(0^{\circ m}/90^{\circ}/0^{\circ})$ laminated magnetostrictive shell by using the GDQ method. The amplitude of center displacement $W(L/2)$ is large when without velocity feedback control gain value $k_c c(t)$ = 0. We find that with velocity feedback control and with suitable values $k_c c(t)$ = 10^6 can reduce the amplitude of displacement to a smaller value. **Fig. 3** shows that transient responses of dominant thermal stress $\overline{\sigma}_\theta = \sigma_\theta / E_2$ on $Z = -1/6$, $X = 1/2$ under sudden uniform input: $T_0(\frac{1}{2},t)$, $q_0 = 2\,{}^{Btu}\!/_{\sec\cdot in^2}$, $\theta = 1$ radian, N = 73, for $(0^{\circ m}/90^{\circ}/0^{\circ})$ laminated magnetostrictive shell by using the GDQ method. The amplitude of thermal stress $\overline{\sigma}_\theta |_{Z=-1/6, X=1/2}$ is large when without velocity feedback $k_c c(t)$ = 0. We find that with velocity feedback control and with suitable values $k_c c(t)$ = 10^6 can reduce the amplitude of thermal stress $\overline{\sigma}_\theta$ to a smaller value.

5. Conclusions

In the study of transient responses of the rapid heating in three-layer laminated magnetostrictive shell, the computational GDQ method provides a method for calculating the displacements and stresses. GDQ solutions show that: With velocity feedback control and with suitable values in laminated magnetostrictive shell can reduce the amplitudes of displacement and stress to smaller value, respectively.

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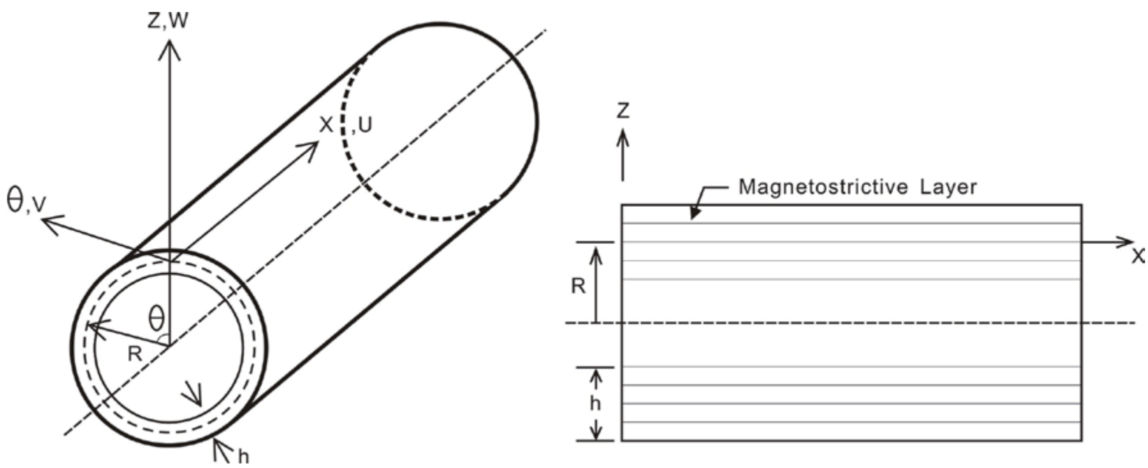

Fig. 1 Laminated magnetostrictive shell.

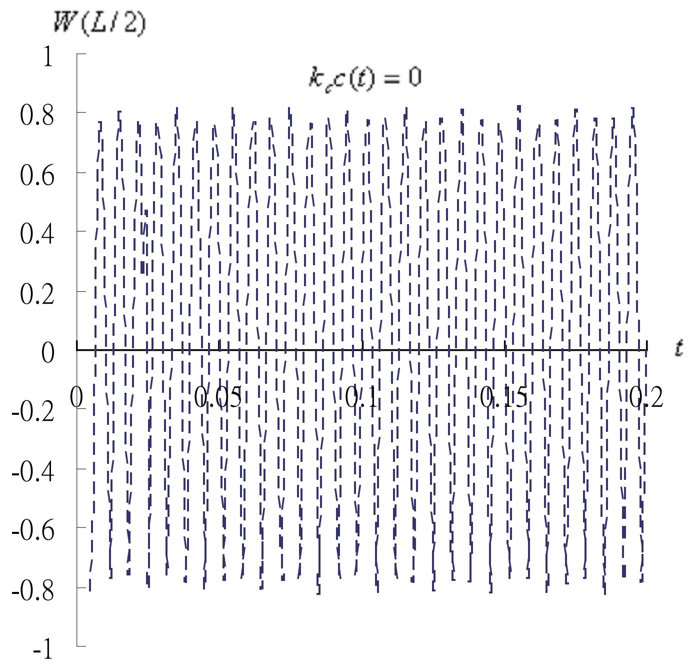

Fig. 2(a) control gain value $k_c c(t) = 0$

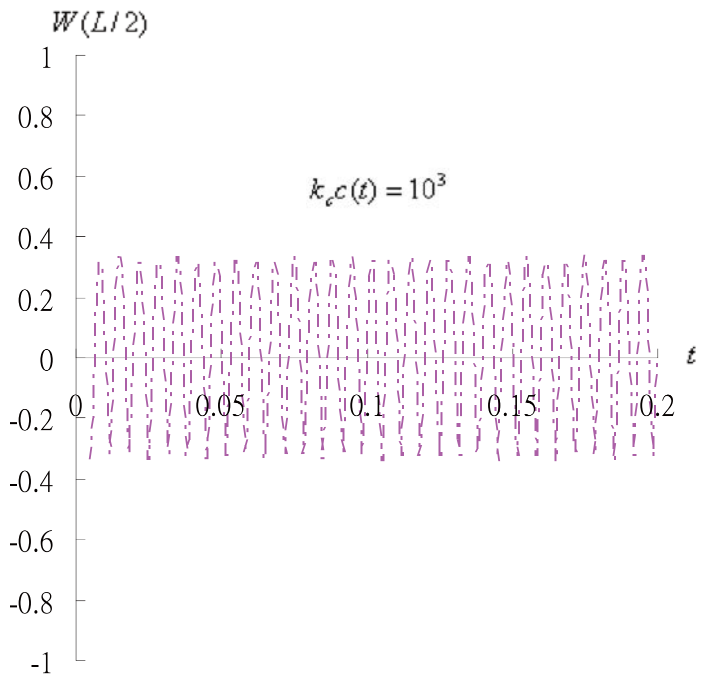

Fig. 2(b) control gain value $k_c c(t) = 10^3$

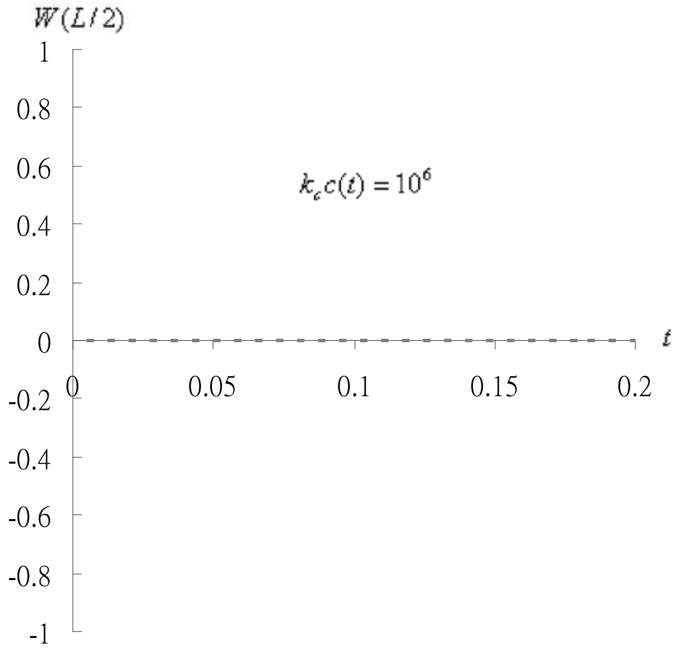

Fig. 2(c) control gain value $k_c c(t) = 10^6$

Fig. 2 $W(L/2)$ vs. t controlled by gain values $k_c c(t)$

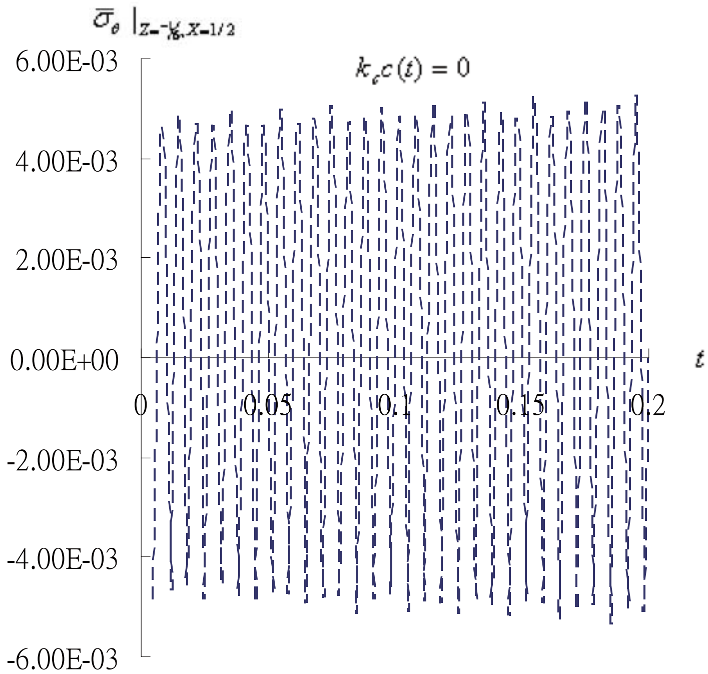

Fig. 3(a) control gain value $k_c c(t) = 0$

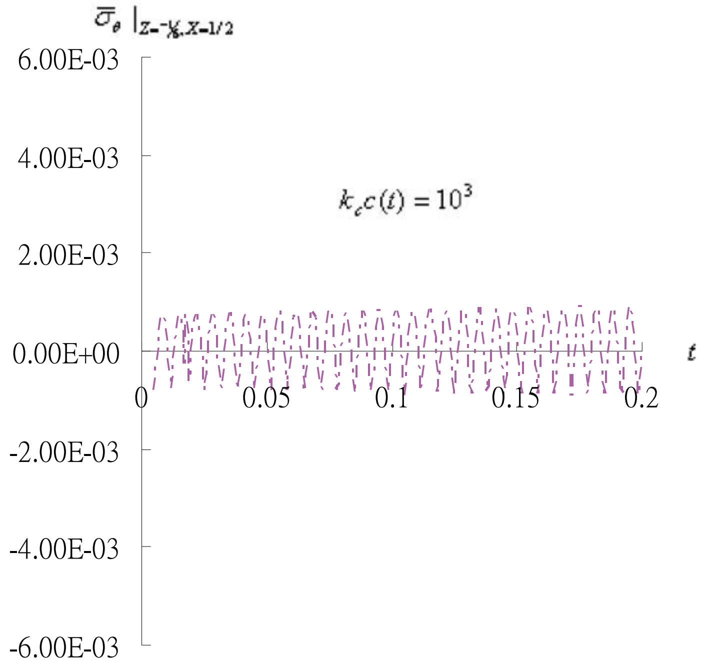

Fig. 3(b) control gain value $k_c c(t) = 10^3$

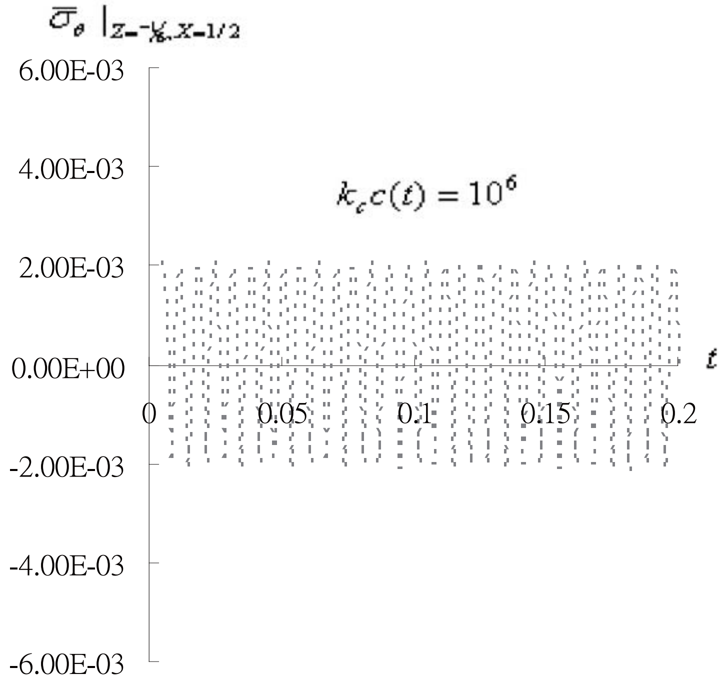

Fig. 3(c) control gain value $k_c c(t) = 10^6$

Fig. 3 $\bar{\sigma}_\theta |_{Z=-1/6, X=1/2}$ vs. t controlled by gain values $k_c c(t)$